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Double-Mirror Method for Rangefinding: A Scientific Development by al-Khāzinī

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ABSTRACT

The use of mirrors for measuring distance and height has its origins in ancient Greek science. In his *Optics*, Euclid demonstrated how the reflection of light could be geometrically employed to determine an object's altitude or range with remarkable precision. During the Islamic Golden Age, scholars adopted and further developed his methods. For example, ʿAbd al-Raḥmān al-Khāzinī (active ca. 1115–1130 CE), in his treatise *On Marvelous Instruments*, not only explained Euclid's single-mirror method but also introduced an innovative Double-Mirror Method. This innovation enabled the simultaneous determination of both height and distance of an object through one observation. Al-Khāzinī's contribution represents a significant step in linking theoretical geometry with practical applications in surveying and engineering within the scientific tradition of the medieval Islamic world. This article examines his methods in detail.

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فاصله سنجی با دو آینه: دستاوردی علمی از خازنی

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کلیدواژه‌ها: اپتیک اقلیدس، خازنی، فی آلات العجیبة، آینه، تخمین فاصله، هندسه عملی، مسافت یاب، علوم اسلامی

چکیده

به‌کارگیری آینه برای اندازه‌گیری فاصله و ارتفاع، ریشه در سنت‌های علمی یونان باستان دارد. اقلیدس در کتاب المناظر با استفاده از روش هندسیِ بازتاب نور، نشان داد چگونه می‌توان با کمک آینه، ارتفاع یا فاصلهٔ اشیاء را تعیین کرد. در دوران تمدن اسلامی، دانشمندان مسلمان این روش را اقتباس و گسترش دادند. در میان آنان، عبدالرحمن خازنی (منجم و ریاضی‌دان معاصر با خیام در قرن پنجم قمری) در رسالهٔ خود با عنوان عنوان فی آلات العجیبة، افزون بر توضیح روش آینهٔ اقلیدس، روشی جدید برای مساحی با دو آینه ابداع کرد. در این روش می‌توان هم‌زمان هم ارتفاع و هم فاصلهٔ جسم را به دست آورد. نوآوری خازنی گامی مهم در پیوند دادن هندسهٔ نظری با کاربردهای مهندسی و نقشه‌برداری در جهان اسلام به شمار می‌آید.

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I. Proposition 20 of Euclid's *Optics*: the Main Idea of al-Khāzinī's Method

The first Arabic translation of *Optics* was done during the reign of the Abbāsīd Caliph al-Ma'mūn (813-833 A.D.) (Kheirandish, 1998, p. xx).[1] The core idea of Euclidean optics is the equality of the angles of incidence and reflection. Among the other topics, Euclid formalized the geometric principles underlying what we call the Mirror Method. He demonstrated how "mirror" could be used to determine the height or distance of an object.

In Proposition 20, Euclid explains how to determine an object's height by placing a mirror between the observer and the object. This setup allows the mirror to reflect the visual beam emitted from the surveyor's eye to the top of the object (Kheirandish, 1998, pp. 58-61):

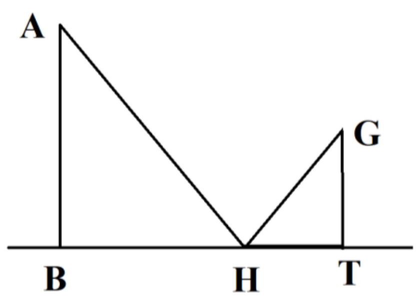

Fig. 1. Proposition 20 of Euclid's *Optics*.

Let the object be *AB*, and the observer's eye be at point *G*. Place a mirror *H* on the ground so it is aligned straight toward the base of the object at *B* (Fig. 1). Draw a visual ray from the eye *G* toward the mirror, hitting it. Let the reflected ray bounce until it reaches the top of the object at *A*. From point *T*, draw a straight line along *BH*, called *HT*. From *G*, draw a line perpendicular to *HT*, called *GT*. Because the visual ray *GH* strikes the mirror at *H* and reflects toward *A*, the angle of incidence equals the angle of reflection. Therefore, $\angle GHT = \angle AHB$, and since

1. The earliest known manuscript of the Arabic translation of *Optics* dates back to 1203 (Kheirandish, 1998, p. xxvi).

$\angle ABH = \angle GTH$, the remaining angle $\angle HGT$ equals $\angle HAB$. According to the Thales theorem, we have:

$$AB = \frac{BH \times GT}{HT} \quad (1)$$

II. Al-Khāzinī: *On Marvelous Instruments*

Abū al-Fath ʿAbd al-Raḥmān Manṣūr al-Khāzinī (active between 1115 and 1130) was among the most renowned astronomers of the 12th century. Originally a Greek slave who later embraced Islam, he served at the court of Sultan Sanjar (who ruled from 1118 to 1157). His major work, *Mīzān al-Ḥikma* (*The Balance of Wisdom*), is a comprehensive manual detailing practical techniques for determining the specific weight of metals and gems. The Greek translation of his astronomical tables, *Zīj al-Muʿtabar al-Sanjarī*, played an important role in the Byzantine revival of astronomy (Abattouy, 1997, pp. 480–481).

His short treatise *On Marvelous Instruments* (*fī Ālāt al-ʿAjība*), describes several astronomical instruments. He explains the instruments of his time and how they could also be applied to land surveying (Kiani Movahed, 2019, p. 189). He detailed how to utilize these instruments and their reading for determining the distance to a specific location or the height of a minaret, tree, hill, etc. Although he does not explicitly cite Euclid, the fifth book of his treatise suggests that he was familiar with the Euclidean optical tradition, possibly through indirect transmission rather than a direct Arabic translation.

II.I. Structure of On Marvelous Instruments

On Marvelous Instruments has a brief preface, seven main books (*maqālah*), and an appendix. The Books 1–4 describe astronomical devices, how to build them and how to use them. These instruments are Triquetrum, Dipotra, Triangle Instrument, and Quadrant. The Books 5–7 cover techniques for finding distances and heights, mainly for architecture and surveying by Mirror, Astrolabe, and Simple Tools. Each book includes three parts (*qism*): construction of the instrument, its application, and geometric proof of the methods. These parts are subdivided into chapters (*bāb*) and sometimes into smaller sections (*faṣl*). After finishing the seventh book, Khāzinī adds an appendix with methods for estimating the distance and size of enemy forces and other military uses by Stick Method (Kiani Movahed, 2019, pp. 190-191).

III. Al-Khāzinī's Rangefinding Method by Mirror

In his fifth book[1], al-Khāzinī outlines three problems related to determining an object's distance or height using Mirror Method (Al-Khāzinī, pp. 20-22):

P1: Determining the height of an object using a single mirror (Single-Mirror Method).

P2: Determining the distance to an object when its height is known (Single-Mirror Method).

P3: Simultaneously determining both the height and distance of an object using two mirrors (Double-Mirror Method).

III.I. Al-Khāzinī's Single-Mirror Method for Altimetry

P1 corresponds to Euclid's the proposition 20 of *Optics*. Rather than repeating Euclid's construction, al-Khāzinī restates the procedure in a more practical language[2] (Al-Khāzinī, pp. 20-21):

> **Part I. On How to Use the Reflection**
> To do this, we place a flat mirror or a vessel filled with water on the ground. Then, we move away from it in the opposite direction of the object being observed, such as a minaret, wall, or mountain apex, until we reach a position where the top of the minaret or wall is visible in the mirror. Once we reach this point, we measure the distance between our standpoint and the mirror, and we call this measured distance the "absolute distance." This is what we intended to describe (Fig. 2).
>
> **Part II. On Deriving [Terrestrial] Distances**
> **Chapter I.**
> **Section I. When the Height of the Object is Unknown, and the Distance Between the Mirror and the Object is Known**
> To determine this, we multiply the known distance by the surveyor's height from the eye to the ground and divide the result by the absolute distance. This yields the height of the minaret, wall, or mountain.

1. The original Arabic text of "fifth book" is introduced as appendix (Text 1-5) in this article.
2. Arabic text in Appendix: Text 1

Al-Khāzinī introduces the proof in the last part of the fifth book[1] (Al-Khāzinī, p. 21):

Part III. Proofs of Chapter I of this Book

Chapter I. Proofs of Section I and Section II of Chapter I[2]

Let *AB* be a minaret perpendicular to the ground, and let *E* be the mirror's center (Fig. 2). We move away from it along the line *EB* until we reach a position where the top of the minaret *A* is visible at the center of the mirror *E*. The standpoint is the point *D*. The surveyor is at point *G*, and *GD* is the surveyor's height from the ground to the eye. Since the angles *AEB* and *GED* are equal, the triangles *AEB* and *GED* are similar. Therefore, the ratio of *AB* (the height of the minaret) to *BE* (the distance from the mirror to the foot of the minaret) is equal to the ratio of *GD* (the surveyor's height) to *DE* (the distance between the surveyor and the mirror, which we called the absolute distance). [As a result, either *AB* or *BE*, if unknown, can obtained from proportion.]

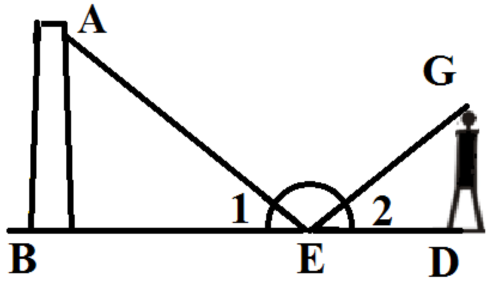

Fig. 2. Single-Mirror Method for Altimetry and Planimetry.

By using modern notions:

$\triangle ABE \simeq \triangle DGE$ (2)

$\frac{AB}{BE} = \frac{DG}{GE}$ (3)

$AB = \frac{BE.DG}{GE}$ (4)

While the Mirror Method is straightforward, it has limitations. The drawback of this method is that points *B*, *E*, and *D* must all lie in a vertical plane, and the

1. Arabic text in Appendix: Text 4

2. This proof belongs to both P1 and P2.

terrain must be flat. Measuring the height (*AB*) becomes impossible if the surveyor cannot stand on a straight line along *BE*.

III.2. Al-Khāzinī's Single-Mirror Method for Planimetry

Al-Khāzinī solves P2 by adapting Proposition 20 to the unknown distance[1] (Al-Khāzinī, p. 21):

> **Section II. When the Distance Between the Mirror and the Base of the Object is Unknown, but the Height of the Object is Known**
> To determine this, we multiply the object's height by the absolute distance and divide the result by the surveyor's height. This gives the distance.
> The surveyor determines the distance from the mirror to the object's foot by knowing or estimating the height of the object and measuring the distance from his position to the mirror[2] (Al-Khāzinī, p. 21).

According to Proposition 20, and by using modern notions (Fig. 2):

$\triangle ABE \simeq \triangle DGE$ (5)

$\frac{AB}{BE} = \frac{DG}{GE}$ (6)

$BE = \frac{AB.DE}{DG}$ (7)

However, the previously mentioned drawbacks still apply in this case. Additionally, measuring distances may prove impossible due to obstacles in the terrain. For example, if the object is located on the apex of a hill (Fig. 3), the surveyor cannot accurately measure the distance to the object (*BE*) due to the curvature of the hill. Nonetheless, such methods are effective in areas like plains or deserts.

1. Arabic text in Appendix: Text 2
2. Arabic text in Appendix: Text 4

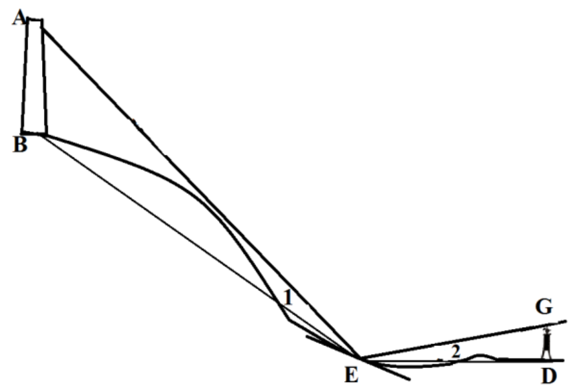

Fig. 3. The drawback of measurement of the height or distance by Mirror Method.

III.3. Al-Khāzinī's Double-Mirror Method

P3 is a compound problem, al-Khāzinī needs to propose an innovation to solve it. He suggests using two mirrors when the distance to an object is not easily accessible, such as when the object is located on the opposite bank of a river. This approach allows the surveyor to determine both the distance and height of the object simultaneously[1] (Al-Khāzinī, pp. 21-22):

> **Section III. When Both the Distance and the Height of the Object are Unknown**
> If the minaret's foot is inaccessible or the object is a mountain apex obscured by foliage or mist, we place the mirror and derive the first absolute distance. Then, we recede a known distance along the same line and place another mirror, deriving the second absolute distance. We subtract the first absolute distance from the second and call the remainder the "difference." Next, we multiply the distance between the two mirrors by the first absolute distance and divide the result by the difference. This gives the distance between the first mirror and the object's foot. We then multiply this by the surveyor's height and divide the total by the first absolute distance, yielding the height of the minaret.

Since al-Khāzinī does not cite a source for this procedure, the method likely reflects practical expertise of architects and engineers in his time. Again, al-

1. Arabic text in Appendix: Text 3

Khāzinī introduces his proof in the last part of the fifth book[1] (Al-Khāzinī, pp. 21-22):

Chapter II. Proof of Section III of Chapter I

Let *AB* be the minaret, *E* the center of the mirror at the first standpoint, and *D* the first standpoint where *ED* is the first absolute distance along the line *EB*. Let *T* be the second mirror and *L* the second standpoint where *A* is visible at *T*, with *LT* being the second absolute distance. We draw line *ZGKS* parallel to the ground and perpendiculars *ZṢ* and *SO*, transforming triangles *GDE* and *KLT* into triangles *ZEṢ* and *SOT*, [respectively]. We draw *SM* parallel to *ZE*, making triangles *ZEṢ* and *SMO* equal. In the similar triangles *ATE* and *STM*, the ratio of *TM* to *TE* equals the ratio of *MS* (or *EZ*) to *EA*. In the similar triangles *ABE* and *EZṢ*, the ratio of *EZ* to *EA* equals the ratio of *EṢ* to *EB*. Therefore, the ratio of *TM* (the difference) to *TE* (the distance between the mirrors) is equal to the ratio of *EṢ* (the first absolute distance) to *EB* (the distance between the first mirror and the foot of the minaret). The ratio of *EB* to *BA* (the desired height) is equal to the ratio of *EṢ* ([the first absolute] distance) to *ṢZ* (the surveyor's height). This is what we intended to explain.

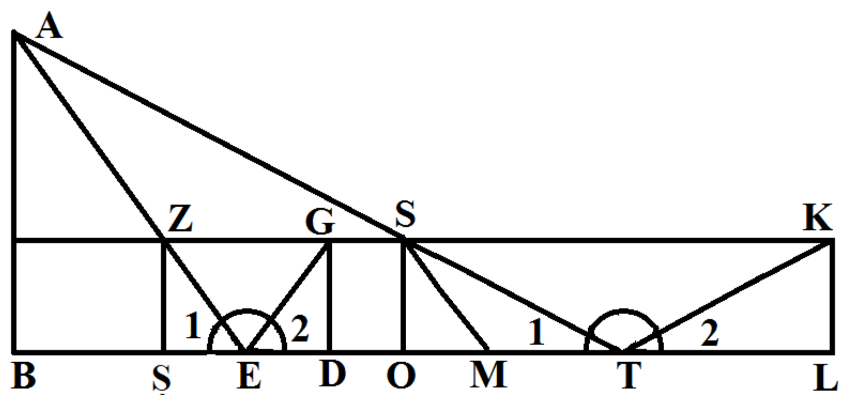

Fig. 4.The modern figure of the Double-Mirror Method.

Al-Khāzinī elaborates on a technique to simultaneously determine both the distance and height of the object using two mirrors. In this case, *AB* shows the object, *E* and *T* denote the first and second mirrors, respectively. The surveyor should position himself at point *D* to view the object's apex in the first mirror

1. Arabic text in Appendix: Text 5

(Fig. 4). Subsequently, the surveyor must move back to point *L* to see the object's apex in the second mirror. By measuring the surveyor's height (*GD*) and the distance between the two mirrors (*ET*), one can derive the necessary calculations:

$\triangle DGE = \triangle Z\d{S}E$ (8)

$\triangle KLT = \triangle SOT$ (9)

If *SM* is drawn parallel to *ZE* from point *S*:

$\triangle SMO = \triangle Z\d{S}E$ (10)

$TM = OT - OM = TL - OM = TL - ED$ (11)

$\triangle ATE \simeq \triangle STM$ (12)

$\frac{TM}{TE} = \frac{SM}{AE}$ (13)

$AE = \frac{TE.SM}{TM}$ (14)

$\triangle ABE \simeq \triangle Z\d{S}E$ (15)

$\frac{EZ}{AE} = \frac{\d{S}E}{BE}$ (16)

(14) & (16) $BE = \frac{AE.\d{S}E}{EZ} = \frac{TE.SM.\d{S}E}{TM.EZ} = \frac{TE.\d{S}E}{TM} = \frac{TE.ED}{TL-ED}$ (17)

After obtaining *BE*, one can calculate the object's height (*AB*):

(15)→ $AB = \frac{BE.\d{S}Z}{CE} = \frac{BE.GD}{ED}$ (18)

(17) & (18)→ $AB = \frac{TE.ED.GD}{ED(TL-ED)} = \frac{TE.GD}{TL-ED}$ (19)

IV. Conclusion

In his *Optics*, Euclid presented the use of a mirror as an instrument for determining the height of an object through a geometrical procedure. Al-Khāzinī, in *On Marvelous Instruments*, advanced Euclid's method in two cumulative stages. The first modification involved retaining the original apparatus while transposing the unknown variable from height to distance. In the second stage, al-Khāzinī introduced an innovative second mirror within a modified configuration. This method allowed him to resolve the problem for both height and distance concurrently. Al-Khāzinī's Double-Mirror Method functions as an early form of indirect range-finding.

Thus, the historical development from Euclid to al-Khāzinī demonstrates not only the continuity of scientific reasoning but also the gradual shift from theoretical optics toward instrument-based measurement in practice.

Appendix

The edition ofal-Khāzinī's original text is based on the four known :manuscripts (Kiani Movahed, ۲۰۱۹' p.۱۹۰)

- M: manuscript No. ۲/۶۴۱۲' Majlis Library, Iran
- S: manuscript No. ۱/۶۸۱' Sepahsālār School Library, Iran
- I: manuscript No. A.Y-۳۱۴' Istanbul Library, Turkey
- Q: manuscript No. ۴۵H.K ۴/۶۵۹۱' Manisa Library, Turkey

The Istanbul manuscript was reproduced as a facsimile by Fuat Sezgin in his collection. Nevertheless, none of these manuscripts had undergone critical .emendation prior to the completion of Kiani Movahed's thesis for masters degree

The Sepahsālār manuscript (S) is used as the base text because it is the most complete and least defective, and the best handwriting. Differences with the Majlis (M), Istanbul (I), and Manisa (Q) manuscripts are recorded in footnotes. In order to enhance the clarity of the Arabic text, we supplied certain supplementary words enclosed in [].

.Text ۱

المقالة الخامسة: في معرفة المسافات بانعكاس من الشعاعات البصريّة

وإذ قد بيّنّا في المقالة الرابعة شرح الآلة المعروفة بالسدس. فالآن نخوض في هذه المقالة بذكر طرق توصل إلى استخراج المسافات وارتفاعات الأشياء بالآلة المعروفة بالمرآة أو ما ناب عنها من الماء أو غيره من المايعات الّتي تنعكس عنها الشعاعات البصريّة، ونسأل الله التوفيق.

بعد ما يسلّمنا عن القدماء أن زوايا انعكاسات الشعاعات من سطوح الأشياء الصقلة مساويةٌ لزوايا مجيء تلک الشعاعات، فنبني هذه المقالة على تلک المقالة وهي تشتمل على ثلاثة أقسام: الأوّل في كيفيّة استعمالها، الثاني في استخراج الأبعاد، الثالث في البرهان عليها. ونذكر كلّ قسم منها مفصّلاً إن شاء الله تعالى.

القسم الأوّل: في كيفيّة استعمالها

إذا أردنا ذلک نضع على وجه الأرض مرآةً مستويّة الوجه أو آنيةٌ من الأواني فيها ماءٌ. ثمّ نبعد عنها إلى خلاف جهة الشيء المنظور إليه من منارةٍ أو حائطٍ أو رأس جبلٍ إلى أن نبلغ إلى موضعٍ يري في المرآة رأس المنارة

أو الحائط. فإذا انتهينا، ذرعنا ما بين موقفنا إلى المرآة وسمّينا ما وجدناه من الذراع البعد المطلق. وذلک ما أردنا أن نصف.

القسم الثاني في استخراج المسافات [الأرضيّة]

الباب الأوّل

الفصل الأوّل إذا كان ارتفاع الشيء المنظور إليه مجهولاً وكانت المسافة بين المرآة والشيء المنظور إليه معلومة

إذا أردنا معرفة ذلک، ضربنا عدد المسافة في عدد قامة الناظر من العين إلى الأرض، وقسّمنا المبلغ على البعد المطلق، يخرج طول المنارة أو الحائط أو الجبل المطلوب.

.Text ٢

الفصل الثاني: إذا كانت المسافة بين المرآة وأصل الشيء المنظور إليه مجهولة وارتفاع الشيء المنظور إليه معلوماً[١]

إذا أردنا معرفة ذلک، ضربنا ارتفاع الشيء المنظور إليه في البعد المطلق وقسّمنا المبلغ على قامة الناظر، فيخرج المسافة.

.Text ٣

الفصل الثالث: إذا كانت المسافة وارتفاع الشيء المنظور إليه كلاهما مجهولين

إذا كانت المنارة غير موصول إلى أصلها أو المطلوب[٢] عمود جبلٍ في داخله مغشى بذبولةٍ وسفوحةٍ، نضع المرآة ونستخرج البعد المطلق الأوّل. ثمّ نتباعد[٣] على ذلک السمت تباعداً معلوماً، ونضع مرآة أخرى ونعلم البعد المطلق الثاني. ثمّ ننقص البعد الأوّل من الثاني، ونسمّى الباقي الفضلة. ثمّ ضربنا أذرع مابين المرآتين

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١. Q: وارتفاعه معلوماً

٢ I: مطلوب

٣ M: تباعدنا

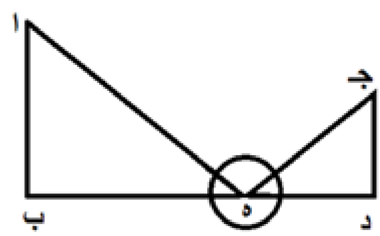
في البعد المطلق الأوّل، وقسّمنا المبلغ على الفضلة، فيخرج ما بين المرآة الأولى وبين أصل الشيء المنظور إليه. فتضربه في طول القامة [الناظر]، ونقسّم المجتمع على البعد الأوّل المطلق، فيخرج طول المنارة.

.Text ۴

القسم الثالث: في البرهان على الباب الأوّل من هذه المقالة

الباب الأوّل: في البرهان على فصلي[1] الثاني و الثالث[2] من الباب الأوّل

فليكن **اب** منارةً قائمةً على سطح الأرض والمرآة هي الّتي مركزها **ه**. ونبعد عنها على سمت خطّ **ه ب**[3] إلى أن نبلغ[4] موضعاً يري فيه ا رأس المنارة على **ه** وسط المرآة وهو **د**؛[5] والناظر فيها هو **جـ** و**جـ د** قامة الإنسان الناظر مابين[6] الأرض والبصر. ولأنّ تساوي زاويتي[7] **اهب** [و] **جـ ه د** يلقّي التشابه بين[8] مثلّثي **اهب** [و] **جـ ه د**، فتكون نسبة **اب**، طول المنارة، إلى **ب ه**، طول مسافة[9] المرآة عن أصلها، كنسبة **جـ د**، ارتفاع الناظر بمقدار القامة، إلى **د ه**، مابين الموقف وبين المرآة، وهو الّذي سمّيناه البعد المطلق.



.Text ۵

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[1] Q: فصل

2. ''الفصل الأول'' and ''الفصل الثاني'' are correct, but Arabic text says: ''الفصل الثاني'' and '' الفصل الثالث''.

۳. M: ه ک

۴. Q: يبلغ

۵. Q: ب

۶. Q: هي

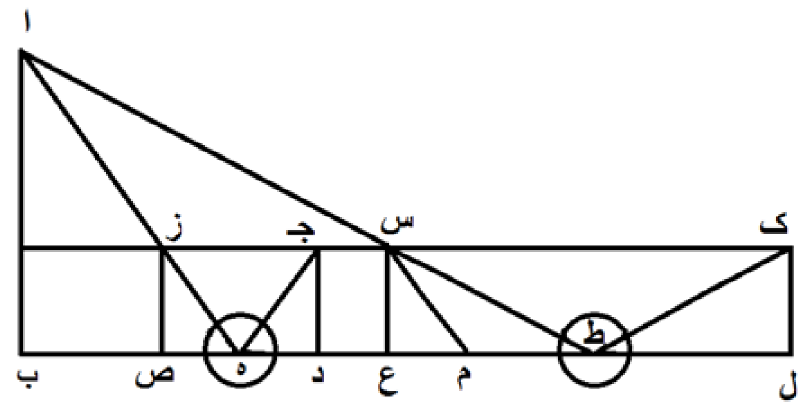
۷. Q: -زاويتي

۸. Q: هي

۹. M: -المسافة

الباب الثاني: في البرهان على الفصل الرابع[1] من الباب الأوّل

فليكن **ا ب** المنارة و**ه** مركز المرآة عند الموقف الأوّل و**د**[2] الموقف الأوّل و**ه د** البعد الأوّل المطلق على مسامتة **ه ب**.[3] **ط** المرآة الثانية و**ل** الموقف الثاني الّذي يري منه **ا** على **ط** و**ل ط** البعد الثاني. ونخرج[4] **ز جـ ك س** على موازاة الأرض وأخرجنا عمودي **ز ص** [و] **س ع**، يحوّل مثلّثا **جـ د ه** [و] **ك ل ط**[5] إلى مثلّثي **ز ه ص** [و] **س ع ط**. ويخرج **س م** موازيّاً لـ**ز ه**، فيتساوي مثلّثا **ز ه ص** [و] **س م ع**. وفي مثلّثي **ا ط ه** [و] **س ط م** المتشابهين، نسبة **ط م** إلى **ط ه** كنسبة **م س**، المساوي لـ**ه ز**، إلى **ه ا**. و[في مثلّثي **ا ه ب** و **ه ز ص** المتشابهين] نسبة **ه ز** إلى **ه ا**[6] هي نسبة **ه ص** إلى **ه ب**. فنسبة **ط م**، الفضلة، إلى **ط ه**، ما بين المرآتين، كنسبة **ه ص**، البعد الأوّل المطلق، إلى **ه ب**، المسافة بين المرآة الأولى وبين أصل المنارة، ونسبة **ه ب** إلى **ب ا**، المطلوب، كنسبة **ه ص**، البعد، إلى **ص ز**، القامة [الناظر]، وذلک ما أردنا بيانه[7].



1. "الفصل الثالث" is correct, but Arabic text says: "الفصل الرابع".

٢. M: جـ

٣. Q: ه ط

٤. Q: ويخرج

٥. M: ک ا ط

٦. M: -و نسبة ه ز إلى ه ا

٧. Q: نبيّن

Bibliography

Abattouy, Muhammed. (1997). "Al-Khāzinī." *Encyclopedia of the History of Science, Technology, and Medicine in Non-Western Cultures,* Springer.

Al-Khāzinī, ʿAbd al-Raḥmān. (n.d.). *Risāla fī al-Ālāt al-ʿAjība* (*On Marvelous Instruments*). Manuscript, Sepahsālār Mosque Library (Tehran).

Kheirandish, Elahe. (1998). *The Arabic Version of Euclid's Optics.* (First edition.) Vol. 1 New York City: Springer-Verlag.

Kiani Movahed, R. (2019). "ʿAbd al-Raḥmān al-Khāzinī's treatise *Fī Ālāt al-ʿAjiba*: A short Review. "*Journal for the History of Science*, 187-214.

Sezgin, Fuat. (2001). *Manuscript of Arabic Mathematical and Astronomical Treatises.* Vol. 66. Frankfurt: Institute for the History of Arabic-Islamic Science.