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A Persian Translation of al-Jawharī's Additions to *Elements*, Book V

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Abstract
During the third/ninth century, several commentators on Euclid's *Elements* composed blocks of propositions (called *ziyādāt*) in which they expanded specific concepts or techniques beyond the basic discussions of Euclid. Among the earliest of these *ziyādāt* were three propositions created by al-'Abbās b. Sa'īd al-Jawharī in an attempt to demonstrate the validity of Euclid's definitions V, 5 and V, 7. A unique manuscript of a Persian translation of these propositions has recently been discovered. In this paper, I introduce this Persian text, offer an English translation, and discuss the relationship between the Persian and Arabic versions.

Keywords: Jawharī, Euclid, proportion

Introduction

In this paper, we are concerned with the Persian translation of three propositions added by al-Jawharī to book V of the *Elements*. These *ziyādāt* (additions) were originally produced in Arabic (De Young, "Additions …" 153-178)[1] and may have once been a part of al-Jawharī's now lost Arabic commentary on the *Elements*. They were intended to explain Euclid's definitions of "being in the same ratio".

The Manuscript

1. The edition of the Arabic text was based on five manuscripts: Istanbul, Feyzullah 1359/4; Tunis, Aḥmadiyya 5482 (now BN 16167/2); Tehran, Dāniškada-i Adabiyyāt Ğ 284/1; Hyderabad, Osmania University Lib. 483; Princeton University Lib., Yehuda 4850. A sixth Arabic manuscript has now come to light: British Library, DEL. AR 1909/c (fols. 166a-167b). This newly discovered manuscript is copied in a clear *nasta`līq* and lacks the diagrams that normally accompany the text.

The Persian rendition of al-Jawharī's *ziyādāt* (added propositions) is found on two interleaved sheets bound into manuscript Tehran, Sanā 226 following the standard Euclidean definitions of book V. This Persian translation occupies the recto and verso of the first sheet and the recto of the second (its verso is blank). These sheets are approximately five centimeters shorter vertically than the manuscript pages, and contain 17-20 lines of text per page. The text begins with the standard "Bismillah" or pious invocation, indicating that it began its life as a relatively independent unit and not merely as pages extracted from a larger treatise. There is no colophon for these additions and the text of the translation cannot be dated from internal evidence, although the hand of the copyist appears to be relatively recent. The author's name is given as al-'Abbās ibn Sa'īd, as in the Arabic original.

Tehran, Sanā 226[1], is a copy of the Persian translation of Naṣīr al-Dīn al-Ṭūsī's *Taḥrīr Kitāb Uqlīdis* made by Quṭb al-Dīn al-Shīrāzī (De Young, "Quṭb al-Dīn al-Shīrāzī,"). Its colophon is dated 698AH[2]. The treatise has been copied in two hands. The majority of the treatise is copied in a scrawling, coarse *naskhī* with approximately twenty lines per page. There are guide words at the end of each folio to ease the reader's transition to the next. Diagrams are typically positioned in openings along the left-hand margin of the text column. Because of the way text is filled in around the diagrams, it appears likely that they are prepared by the copyist himself during the process of copying the text. I consider this material to represent the work of the copyist himself.

Two lacunae, from the end of proposition V, 10 to the end of proposition VI, 27 and from proposition XI, 37 to proposition XII, 6, have been filled by another hand using a smaller, more precise and

1. The current cataloging gives the number as Sanā 226. Sezgin (114) gives the number as Sanā 227.

2. Apparently this date refers to the date of translation, not the date of the manuscript copy (although the hand of the copyist does appear to be quite old). On the title page, there is a calculation of 710 (the Hijra date of al-Shīrāzī's death) minus 698 (the date of the colophon), showing the remainder to be 12. Presumably this calculation indicates that the translation was completed twelve years prior to the translator's death. Sezgin (114), citing Dāniš Pažūh (485) gives the date of completion as 681 AH, and dates the manuscript to 1072 AH.

somewhat angular *naskhī*. These sections, comprising 19 folios and 13 folios respectively, are copied with wider margins than the major portion of the manuscript, and have usually seventeen lines per page. In these sections there are no guide words to ease the transition to the next folio. Diagrams are typically placed in rectangular openings in the text, positioned on either left or right margin of the column, so near the end of the proposition that the text of the next proposition typically begins immediately beside the diagram of the previous proposition. Since these sections that were copied by the second hand now form a seamless whole with the material copied by the first hand, I think it probable that this material was a replacement for folios that had become separated from the original manuscript. Perhaps they were in the possession of another person and could not be retrieved, necessitating that they be re-copied. Or perhaps, because they had become detached, they had become too tattered to be simply re-inserted into the manuscript.

There are marginal annotations, sometimes extensive, in both portions of the manuscript. Annotations in the original part are mainly in two hands, that of the first copyist and that of the copyist of the lacunae replacements. Annotations in the lacunae replacement sections are almost always in the hand of the replacement copyist and none are in the hand of the first copyist. It seems likely, since the two sections of the manuscript mesh so perfectly, that the copyist who created the replacements deliberately tried to preserve not only the text of the folios that were being recopied (for whatever reason) but also the foliation and marginalia of the "original" manuscript. It is probable that this copyist was also the owner of the "original" manuscript at some time, since marginalia in his hand are found in both sections of the treatise.

The two interleaved sheets containing al-Jawharī's three added propositions have been copied in the same hand as that used to fill in the lacunae. There are numerous other interleaved notes, written on odd-sized pieces of paper, bound into the manuscript. Many (but not all) of these interleaved sheets are also written in the same hand as the two lacunae replacement sections.

The Author

We know little about the life of the author to whom these *ziyādāt* (additions) are attributed. Al-ʻAbbās ibn Saʻīd al-Jawharī was active during the late 2nd and early 3rd century/first half of the 9th century (Sabra, 79-80; Sezgin, 243-244; Brentjes, "Al-Jawharī," 470-472), although just about the only reliable dates associated with his life seem to be that he participated in some astronomical observations in Baghdād at the behest of Caliph al-Ma'mūn in 214 AH/829-830 AD and in Damascus in 217 AH/832-833 AD, according to a report by Ibn Yūnus (Caussin de Perceval, 57 and 167). A report by al-Bīrūnī has al-Jawharī again making astronomical observations in Baghdād in 228 AH/843 AD (Sezgin, 243). Al-Jawharī was not merely a respected mathematician and observational astronomer, however. Ibn al-Qifṭī describes him as the one who designed and created the observational instruments used by the astronomers of al-Ma'mūn (Ibn al-Qifṭī, 219). Although there is little concrete biographical evidence, we can deduce from what evidence exists that al-Jawharī enjoyed the support and patronage of the political leaders of his day.

Most of al-Jawharī's oeuvre is no longer extant (Sezgin, 243). He is credited not only with a *zīj* (astronomical handbook containing tables and instructions for their use[1]) but also with a commentary on the *Elements*, as well as a treatise (perhaps an extract of this commentary) that introduced some added propositions or *ziyādāt* to book I of the *Elements* (Ibn al-Qifṭī, 219). The only thing we know for certain about these additions is that they included his "demonstration" of the fifth postulate (Jaouiche, 37-44 and 153-178) and at least one additional proposition (al-Ṭūsī, II, *Risāla* 8, 15). Neither of these additions has survived independently, but they are quoted by Naṣīr al-Dīn al-Ṭūsī in the introduction to his demonstration of Euclid's fifth postulate[2].

1. For a brief introduction to *zīj*, see Mercier, 1057-1058.
2. The text of the added proposition is not quoted in its entirety, but al-Ṭūsī gives its enunciation so we know the intent of the proposition (al-Ṭūsī, II, *Risāla* 8, p. 15):

The Euclidean Definitions V, 5 and V, 7

The three propositions that make up this small "addition" or *ziyāda* to book V are a discussion of definitions 5 and 7 of the Euclidean text. These two Euclidean definitions were consistently difficult for readers to understand. I give here the text of these definitions, following the Standard English translation (Heath, II, 114):

(5) Magnitudes are said to be in the same ratio, the first to the second and the third to the fourth, when, if any equimultiples whatever be taken of the first and third, and any equimultiples whatever of the second and fourth, the former equimultiples alike exceed, are alike equal to, or alike fall short of, the latter equimultiples respectively taken in corresponding order

(7) When, of the equimultiples, the multiple of the first magnitude exceeds the multiple of the second, but the multiple of the third does not exceed the multiple of the fourth, then the first is said to have a greater ratio to the second than the third has to the fourth[1].

كل نقطة تخرج منها ثلاثة خطوط مستقيمة فى جهات مختلفة يحيط بثلاث زوايا فالثلاث زوايا معادلة لأربعة قوائم.

"Any point from which three straight lines are extended in different directions, [these lines] bound three angles and the three angles make up four right angles".

1. In modern notation:

A:B = C:D if and only if for all integer multiples mA, mC and nB, nD, we have $mA > nB$ if and only $mC > nD$, $mA = nB$ if and only if $mC = nD$, and $mA < nB$ if and only if $mC < nD$.

The Arabic primary transmission version of these definitions, from Cambridge University Library, ms. *additum* 1036, fol. 51b:

ويقال إن المقادير على نسبة واحدة الأول إلى الثانى و الثالث إلى الرابع إذا كانت الأضعاف المتساوية المأخوذة للأول و الثالث أما زائدة معاً على الأضعاف المتساوية المأخوذة للثانى و الرابع و أما مساوية معاً لها و أما ناقصة معاً عنها إذا أخذت على الولاء أى الأضعاف كانت.

و إذا كانت الأضعاف المتساوية التى أخذت أما أضعاف الأول منها فوائدة على أضعاف الثانى و أما أضعاف الثالث فغير زائدة على أضعاف الرابع فإن نسبة الأول عند ذلک إلى الثانى يقال لها أعظم من نسبة الثالث إلى الرابع.

The Arabic version of these definitions, found in the redaction of al-Ṭūsī, from British Library, ms. add. 23387, fols. 77b-78a:

المقادير التى على نسبة واحدة الأول إلى الثانى و الثالث إلى الرابع هى التى إذا أخذ أى أضعاف أمكن مما لا نهاية لها الأول و الثالث متساوية المرات و الثانى و الرابع متساوية المرات كان الأولين معاً أبداً ما زائدين على الآخرين و أما ناقصين منهما و أما مساويين لهما بشرط أن يؤخذ أى الأول والثالث على الولاء.

فإن كانت مثلاً أضعاف الأول زائدة على أضعاف الثانى و أضعاف الثالث غير زائدة على أضعاف الرابع و لو مرة واحدة بشرط تساوى المرات فى الأول و الثالث و فى الثانى و الرابع كانت نسبة الأول إلى الثانى أعظم من النسبة الثالث إلى الرابع.

These definitions are less than transparent in comparison to most of Euclid's defining statements. The difficulties in trying to understand the definitions were not unique to al-Jawharī. Nor is the difficulty only a matter of linguistics, although the Greek text is certainly difficult to penetrate. Many mathematicians have struggled to clarify Euclid's meaning. Heath, in his notes (2/120-131), outlines the historically more important discussion points relating to these two definitions in a lengthy report.

Al-Jawharī's Added Propositions

The three propositions that al-Jawharī added to book V are intended to (1) show the correctness of Euclid's definition V, 5 (criterion to determine when two ratios are "in the same ratio"), the negation of definition V, 5 (criterion to determine when two ratios are not "in the same ratio"), and Euclid's definition V, 7 (criterion to determine when magnitudes "have a greater ratio" to other magnitudes). Rather, they are more like attempts to explain or clarify what is meant by Euclid's definitions by restating them using the formal linguistic and logical structures of a geometrical proof.

The first proposition substitutes the equimultiples for the original magnitudes and argues that since these equimultiples fulfill the condition of "being in the same ratio" the original magnitudes must do so as well. But this demonstration only substitutes one set of values or magnitudes for another, and asserts that since these new values meet the criterion of

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The definitions of al-Ṭūsī as they were rendered into Persian by al-Shīrāzī, (from Tehran, Sanā 226, unfoliated):

مقادیری که بعض را با بعض نسبتی باشد آنست که ممکن باشد بعض بر بعض زیادت شود بتضعیف مقادیری که بر یک نسبت باشند اول با دوم چون سیوم با چهارم آنست که چون فرا گیرند هر اضعافی ممکن از آنها کی نهایت ندارد اول و ثالث را متساوی المرات وثانی و رابع را متساوية المرات اضعاف اول وسِوم با هم ابدا یا زاید باشند بر اضعاف دوم و چهارم یا ناقص از ایشان یا مساوی ایشان بشرط انک بر ولاء گیرند یعنی اضعاف اول با اضعاف دوم گیرند و اضعاف سیوم با اضعاف چهارم.

پس اگر مثلاً اضعاف اول زاید باشد بر اضعاف ثانی و اضعاف ثالث زاید نباشد بر اضعاف رابع و اگر یکبار باشد بشرط انک مرات در اول و ثالث متساوی باشد و در ثانی و رابع همچنین نسبت اول با ثانی اعظم باشد از نسبت ثالث با رابع.

the definition (itself a kind of *petitio principi* since this fact is simply stated without formal justification) the original values must also meet the criterion. This argument seems to come with an implicit assumption that the principle includes an inherent property of transitivity, that if the argument holds for a specific set of values, it will hold for all values that share the same relationship, including the original magnitudes given in the definition.

The second proposition, using a *reductio ad absurdum* argument, shows that it is impossible for magnitudes that are not "in the same ratio" to produce equimultiples that are "in the same ratio." Al-Jawharī assumes the criterion for "being in the same ratio" when applied to the equimultiples and argues that this should be the equivalent to the original magnitudes "being in the same ratio". Since the problem began with the assumption that the original four magnitudes are not "in the same ratio," it is clear that a contradiction has arisen–and therefore their equimultiples cannot be "in the same ratio". Taken together, these first two propositions seem intended to show that Euclid's criterion represents both a necessary and a sufficient condition for determining when magnitudes are "in the same ratio".

The third proposition again begins by taking equimultiples of four given magnitudes, equimultiples that are not "in the same ratio". Al-Jawharī then claims that since the equimultiples are not "in the same ratio," obviously the original magnitudes also cannot be "in the same ratio"–another *petitio prinicipi*. And if these original magnitudes are not "in the same ratio" to one another, then their ratios must be different, one ratio must be greater than the other.

Arabic and Persian Texts Compared

Two features immediately strike our attention when we compare the Persian translation of al-Jawharī's additions to the Arabic original. The first is that the enunciations of these propositions are not a literal rendition or translation of the Arabic. They are more like paraphrases expressing an equivalent idea. The mathematical content of the proofs themselves, however, more closely parallels the formulation of the argument found in the Arabic.

A second verbal feature that immediately stands out is the stereotypical concluding phrase "*wa huwa'l-maṭlūb*". The same stereotypical concluding phrase is found throughout al-Shīrāzī's Persian translation/edition of al-Ṭūsī's Arabic *Taḥrīr Kitāb Uqlīdis* in which these propositions of al-Jawharī are inserted[1]. This concluding formula is sometimes used at the conclusion of demonstrations in Arabic geometric treatises as well, but not commonly. Its use here is striking because it is not the concluding formulation used in the Arabic originals of these translations. Al-Jawharī, in his Arabic version of the additions, concludes his propositions with the more common Euclidean expression "*wa-dhālika mā aradnā an nubayyin*". Al-Ṭūsī, in his redaction, used "*wa-dhālika mā aradnāhu*," a variant of the standard Euclidean expression. Apparently when these treatises were rendered into Persian, the translator felt free to alter these stereotypical expressions.

Another distinctive stylistic feature of the Persian rendition is that it concretizes the equimultiples of the given magnitudes. The Arabic text of these *ziyādāt* specifies only that we take equimultiples. The Arabic paraphrase of these additions found in the Pseudo-Ṭūsī redaction of the *Elements* (109-111) also couched in these general terms. The Persian version, on the other hand, specifies the specific equimultiples for the first and third and for the second and fourth as ten and five respectively. Use of specific numerical examples is not unknown in the Euclidean tradition, of course. In certain diagrams of some Arabic manuscripts, we find specific numerical values entered. It is generally assumed that these numerical values were intended to assist students studying mathematics to understand the abstract arguments by providing specific illustrative examples. And in some Arabic commentaries, specific numerical examples are included in the text. It is possible that the concretization of

1. The same stereotypical phrase is used also in the later Persian translation with commentary of al-Ṭūsī's redaction by Muḥammed Mahdī Narāqī, dated 1202 (1787/88), Minasian ms. 326, UCLA Library.

these equimultiples in this Persian edition of al-Jawharī's propositions had a similar pedagogical motivation.

We should also note that a porism attached to the second proposition, arguing the converse of the proposition's statement, is found only in the Arabic version. Clearly, judging from these changes made in the text, the Persian translator does not see his task as merely conveying the text from one language into another. He apparently feels free to edit and expand upon the text in various ways as well. The result is more like an edition/paraphrase of the text than a literal translation into Persian.

Among other linguistic peculiarities of the treatise are: (1) the retention of Arabic forms for ordinal numbers, although sometimes Persian numbers are used to name the cardinals, and (2) the Arabic term *burhān* (proof) is typically replaced by the term *dalīl* (reason, evidence). Both are valid loan words from the Arabic, but the term *dalīl* is rarely used in this sense of demonstration or proof in Arabic Euclidean discourse during the classical and medieval period.

Edition Procedures

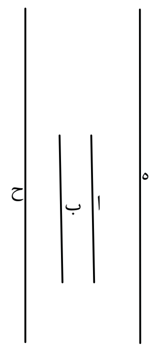
Because Sanā 226 contains the only known copy of this treatise, there are no variants to collect and record. I have transcribed the text as I found it, unless it seemed there was clearly an omission, which is indicated by the square brackets surrounding the supplied material. I have retained the orthography of the manuscript, rather than adopt modernized spellings. Hence I write "زايد" rather than "زائد". Page breaks in the original are indicated with a double slash (//) and the diagrams are redrawn by Microsoft Word2007. In the original manuscript, the text is written as a single block. All paragraphing and formatting in this edition is introduced by the editor to emphasize the logical structure of the treatise.

Persian Text

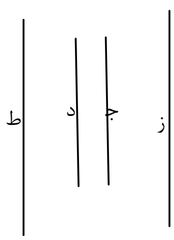
بسم اللّه الرحمن الرحيم

اينرا زيادة كرده است عباس بن سعيد در مقالۀ خامسه از كتاب اقليدس

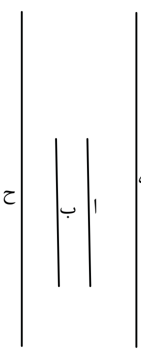
[ا] هرگاه که چهار مقدار متناسب باشد، یعنی نسبت اول با ثانی چون نسبت ثالث باشد با رابع، لازم است که چون اول و ثالث را اضعاف متساوی‌العده فرا گیرند، بر هر وجه که باشد، و ثانی و رابع را بر اضعاف متساوی‌العده فرا گیرند، بر هر وجه که خواهند، اگر اضعاف اول بر اضعاف ثانی زاید باشد، اضعاف ثالث نیز بر اضعاف رابع زاید باشد، و اگر ناقص، ناقص و اگر مساوی، مساوی.



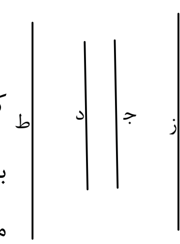
مثلاً نسبت ا با ب چون نسبت جـ است با د. و ا را دَه ضعف گرفتیم مثلاً که آن ه است و جـ را نیز دَه ضعف گرفتیم که آن ز است. و ب را پنج ضعف گرفتیم مثلاً که آن ح است و د را نیز پنج ضعف گرفتیم که آن ط است.پس اگر ه بر ح زاید است، ز نیز بر ط زاید است و اگر ناقص است، ناقص است و اگر مساوی است، مساوی است.



دلیل آنست که چون اربعه متناسب‌اند واجب است که اگر ا بر ب زاید باشد جـ نیز بر د زاید باشد و اگر ناقص، ناقص و اگر مساوی، مساوی. و چون چنین باشد و حال آنک ه دَه ا است و ز دَه جـ است پس اگر ه بر ب زاید باشد ز نیز بر د زاید باشد و اگر ناقص، ناقص و اگر مساوی، مساوی. و چون این معنی مقرر شد و حال آنک ح پنج ب است و ط پنج د است پس اگر ه بر ح زاید است ز نیز بر ط // زاید باشد و اگر ناقص، ناقص و اگر مساوی، مساوی و هو المطلوب.



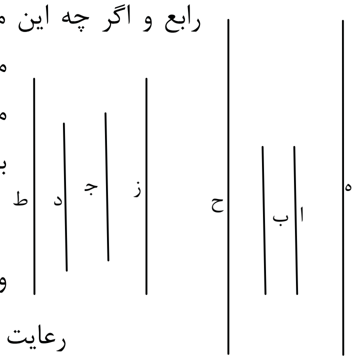
ب؛ و هرگاه که مقادیر اربعه متناسب نباشند حال اضعاف ایشان برین وجه نباشد که گفته شد. زیراکه چون فرض کنیم که نسبت ا با ب همچون نسبت جـ با د نیست بل که ا زاید است مثلاً بر ب و جـ زاید نیست بر د. و ه و ز اضعاف ا و جـ اند بعدهٔ متساویه و ح و ط اضعاف ب و د اند بعدهٔ متساویه چنانک گفته شد پس اگر زانک



هرگاه که ه زاید شود بر ح ز نیز زاید[1] شود بر ط و هرگاه که ناقص، ناقص و هرگاه که مساوی، مساوی.

و حال آنک ه و ز اضعاف متساویةالعدهٔ ا و ج اند که آن ده است مثلاً پس لازم آید که هرگاه که ا زاید باشد بر ح ج نیز [زاید] باشد بر ط و هرگاه که ناقص، ناقص و هرگاه که مساوی، مساوی. و چون چنین باشد و حال آنک ح و ط اضعاف متساویةالعدهٔ ب و د اند [که] آن پنج است مثلاً پس لازم آید که هرگاه که ا زاید باشد بر ب ج نیز زاید باشد بر د و هرگاه که ناقص، ناقص و هرگاه که مساوی، مساوی و این باطل است زیرا که فرض آن بود // که ا زاید است بر ب و ج زاید نیست بر د. پس معلوم شد که هرگاه مقادیر اربعه متناسب نیستند[2] حال اضعاف ایشان بر آن وجه نیست که حال اضعاف اربعهٔ متناسب است و هو المطلوب.

ج ؛ و هرگاه که اضعاف اول زیادت شود بر اضعاف ثانی و اضعاف ثالث زیادت نباشد بر اضعاف رابع و اگر چه این معنی یک کرت باشد بشرط آنک اضعاف اول با اضعاف ثالث متساوی‌العده باشد و همچنین اضعاف ثانی با اضعاف رابع متساویةالعده باشد، واجب است که نسبت اول با ثانی زاید باشد بر نسب ثالث با رابع.



مثلاً ه که اضعاف ا است زاید باشد بر ح که اضعاف ب است و ز که اضعاف ج است زاید نیست بر ط که اضعاف د است با رعایت شرط مذکور. پس واجب است که نسبت ا با ب زاید باشد بر نسبت ج با د.

دلیل آنست که چون نسبت ه با ح اعظم است از نسبت ز با ط و حال آنک ه و ز اضعاف متساویةالعده‌اند مر ا و ج را پس ناچار نسبت ا با ح اعظم باشد از نسبت ج با ط. و چون چنین

۱. زاید (حاشیه).

۲. نیست‌اند (متن).

باشد و حال آنک ح و ط اضعاف متساویةالعده‌اند مر ب و د را پس نسبت ا با ب اعظم باشد از نسبت جـ با د و هو المطلوب.

Translation Procedures

In preparing this translation, I have tried to be literal without allowing the language to become stilted and unintelligible. The paragraphing and formatting follow that of my edition of the Persian text in most cases. Words and phrases added by the translator in order to make the sentence meaning clearer are enclosed in square brackets. Whenever there is a standard mathematical term available in English, I have used that term in place of the circumlocutions that sometimes are used in the medieval Persian. Thus I have used the term "equimultiple" in place of the literal phrase "multiples equal in number."

In the manuscript, each proposition is accompanied by a diagram. These diagrams consist of vertical line segments of differing lengths. The line segments within each diagram are not precisely parallel to one another. This feature is retained in the edited diagrams given here. At the end of most line segments, it appears that the copyist has left a blob of ink – usually extending toward the right-hand side of the line.[1] These blots were presumably intended to accentuate the terminus of the line segment. Or if they are compass prickings, they may be an artifact of the copying process itself. I have not preserved this feature in my edition of the diagrams[2].

These line segments are given letter labels following the usual Arabic *abjad* ordering. These labels appear either beside the line segment or, if

1. Another possibility is that these markings represent the pricking from a compass used to measure out the line segments. The hypothesis that these blots represent compass prickings is attractive, since the pairs of line segments representing magnitudes and equimultiples within each diagram are remarkably uniform in length. But not every line segment has such obvious blots / markings at its termini. Only an examination of the physical manuscript will permit us to reach a firm conclusion on the question.

2. To edit the diagrams, I have used Draft, a software application and associated tool kit developed by Dr. Ken Saito (Osaka Prefecture University). The software is available gratis from Dr. Saito's web site: http://www.greekmath.org/diagrams/. An overview of this software, can be found in Saito (92-94) and the review by De Young ("Draft Software").

the Arabic letter has a significant horizontal feature, they may be placed directly on the line segments. For the sake of legibility, I have displaced any labels found on the line segment to a position beside the line rather than retain their original position. The letter labels are placed approximately at the midpoints of the line segments and are arranged to lie more or less on a straight horizontal line with one another. In general, the segments representing the equimultiples are distinctly longer than the segments representing the initial magnitudes in the proportion.

The ratios of shorter to longer segments range from approximately 3:4 in diagram 1 to a little less than 2:3 in diagram 2 to approximately 3:5 in diagram 3. Whatever the reason for the variability in ratios of lengths in the constructed segments, it clearly does not correspond to the five-fold and ten-fold equimultiples mentioned in the text of the propositions. Evidently the copyist (or his exemplar) did not feel it necessary to make the diagram reflect the specific information included in the verbal component of the proposition.

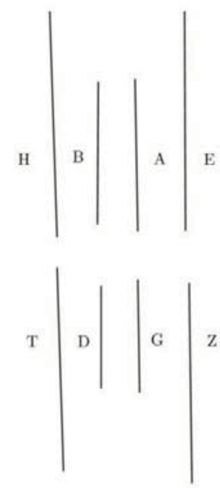
These diagrams are placed in square or rectangular opening in the text column at the end of each proposition. In the case of propositions 1 and 2, the diagram actually follows the text of the proposition, so that it appears alongside the opening text of the next proposition. In my translation, I have retained the placement of the diagrams along the margin as found in the manuscript. However, although I place each edited diagram near the end of each proposition, I keep the diagram within the text area of its proposition, since placement in the style of the manuscript could easily prove confusing to modern readers. I have not attempted to preserve the actual metric of the diagrams, but have scaled them to fit the space. The relative lengths of the line segments and the precise spatial orientations of the diagrams have been retained.

English Translation

In the name of Allah, the Most Merciful, the Most Gracious.

This is the addition which ‘Abbās b. Sa‘īd made to the fifth book of Euclid’s treatise.

[1] Whenever there are four proportional magnitudes, that is, the ratio of the first to the second is like the ratio of the third to the fourth, it is necessary that, when the first and third be given equimultiples, whatever they may be, and the second and the fourth be given equimultiples, whatever they may be, if the multiples of the first exceed the multiples of the second, the multiples of the third exceed the multiples of the fourth; and if less, less; and if equal, equal.



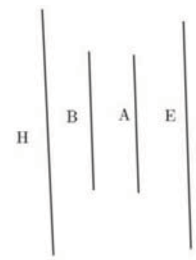
For example, the ratio of A to B is like the ratio of G to D. And A is given a tenfold multiple -let us say, for example, that it is E -and G also is given a tenfold multiple -let us say that it is Z. And B is given a fivefold multiple -let us say, for example, that it is H -and D is also given a fivefold multiple -let us say that it is T. Then, if E exceeds H, Z exceeds T, and if it be less, it is less, and if it be equal, it is equal.

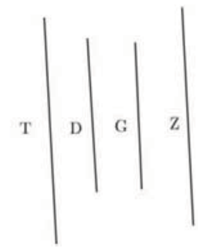
The proof is that since the four[1] are proportional, it is necessary that if A exceeds B, G exceeds D, and if less, less and if equal, equal. And since such is the case, and it is the case that E is ten times A and Z is ten times G, then if E exceed B, Z also exceeds D, and if [it be] less, less and if [it be] equal, equal. And since this statement has been laid down, and it is the case the H is five times B and T is five times D, then if E exceeds H, Z also exceeds T, and if [it be] less, less and if [it be] equal, equal. That is what was sought.

[2] Whenever four[2] magnitudes are not proportional, it is the case that their multiples are not according to the situation mentioned [in the previous proposition]. Because if we specify that the ratio of A to B is not like the ratio of G to D but rather that A exceeds, for example, B and

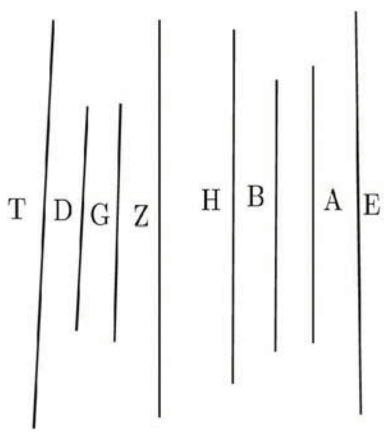
1. It is somewhat unusual that the Arabic term for "four" is preserved here. Earlier, in the enunciation of the proposition, the translator uses Persian term *chahar*.
2. The Arabic term is preserved here.

G does not exceed D and E and Z are equimultiples of A and G and H and T are equimultiples of B and D, as has been said, then, whenever E exceeds H, Z also exceeds T and whenever [it is] less, less and whenever [it is] equal, equal.





And the case is that E and Z are equimultiples, let it be ten, for example, of A and G, then it is necessary that whenever A exceeds H, G exceeds T and whenever [it is] less, less, and whenever [it is] equal, equal. And since such is the case, and the situation is that H and T are equimultiples, let it be five, for example, of B and D, then it is necessary that whenever A exceed B, G also will exceed D and whenever [it is] less, less and whenever [it is] equal, equal. And that is false because the specification was that A exceeds B and G does not exceed D. Thus, it may be known that if four[1] magnitudes are not proportional, the situation of their multiples is not the same as the case of the multiples of the four proportional magnitudes[2]. That is what was sought.



[3] Whenever the multiple of the first exceed the multiple of the second and the multiple of the third do not exceed the multiple of the fourth, even if this holds [only] once [with] the condition that there be equimultiples of the first and the third and likewise equimultiples of the second

1. The Arabic term is preserved here.
2. That is the proportionality of the multiples is dependent on the initial relation of the four magnitudes. If the given four magnitudes are proportional, the multiples are proportional; but if the given magnitudes are not proportional, the multiples also are not proportional.

and the fourth, it is necessary that the ratio of the first to the second exceed the ratio of the third to the fourth.

For example, E which is the multiple of A exceeds H which is the multiple of B and Z which is the multiple of G does not exceed T which is the multiple of D according to the condition mentioned. Thus it is necessary that the ratio of A to B exceed the ratio of G to D.

The proof is that, since the ratio of E to H is greater than the ratio of Z to T and it is the case that E and Z are equimultiples for A and G, then necessarily the ratio of A to H is greater than the ratio of G to T. And since that is the case, and the situation is such that H and T are equimultiples for B and D, then the ratio of A to B is greater than the ratio of G to D. That is what was sought.

Conclusions

As Brentjes has pointed out ("Al-Jawharī," 471), these discussions of al-Jawharī can scarcely be considered truly rigorous demonstrations, since they amount to little more than substituting specific values (the equimultiples) for the general magnitudes and restating the propositions in these more specific terms. But even though they fall short of being rigorous mathematics, they have some historical interest in that they show how difficult these Euclidean definitions were for later mathematicians to comprehend.

We should not discount the role that patronage played in prompting scholars to put pen to paper. Brentjes ("Courtly Patronage," 406-410) has recently pointed out that patronage patterns in Islamic societies changed over time. In the Abbassid period, patronage was largely an individual and complex relationship between a scholar and his wealthy (often politically powerful) patron. Competition for patronage was sometimes fierce and it is probably in that context that we should see the production of many of the rich discussions of Euclid in the Arabic tradition. Such discussions would enable the scholar to show his mathematical prowess by taking on the great Greek mathematician and correcting, adding to, or explicating his work. What better way to demonstrate that one is a worthy of the patron's largess? Since al-Jawharī was personally patronized by al-Ma'mūn and members of his entourage, there is every likelihood that his mathematical writings, including these *ziyādāt*, were at least in part motivated by his need to attract and retain official patronage.

Acknowledgements

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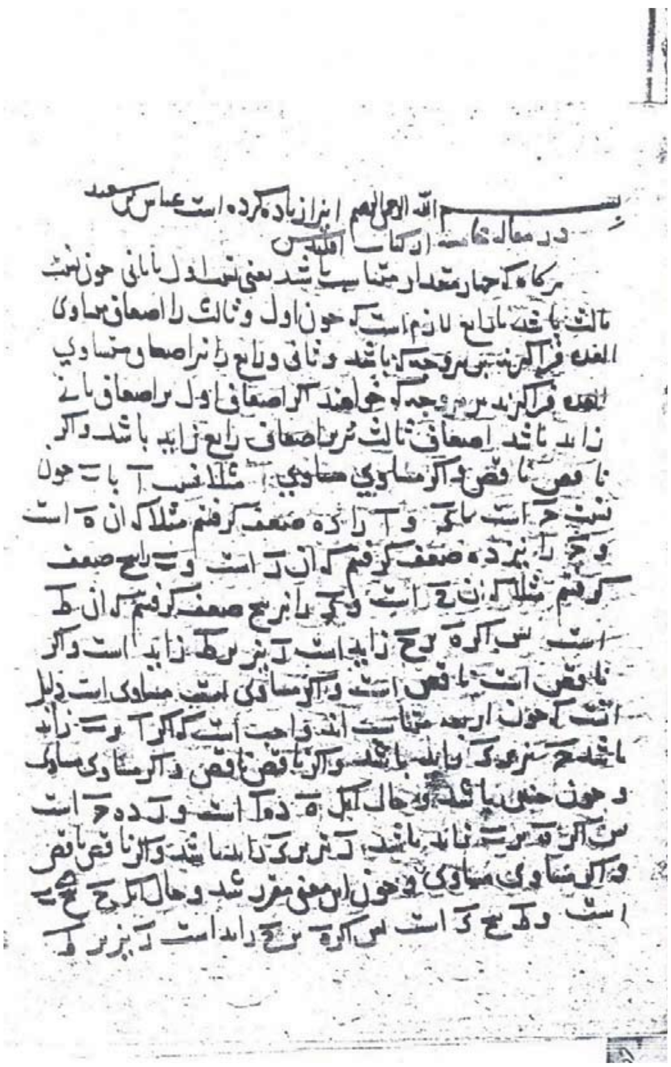

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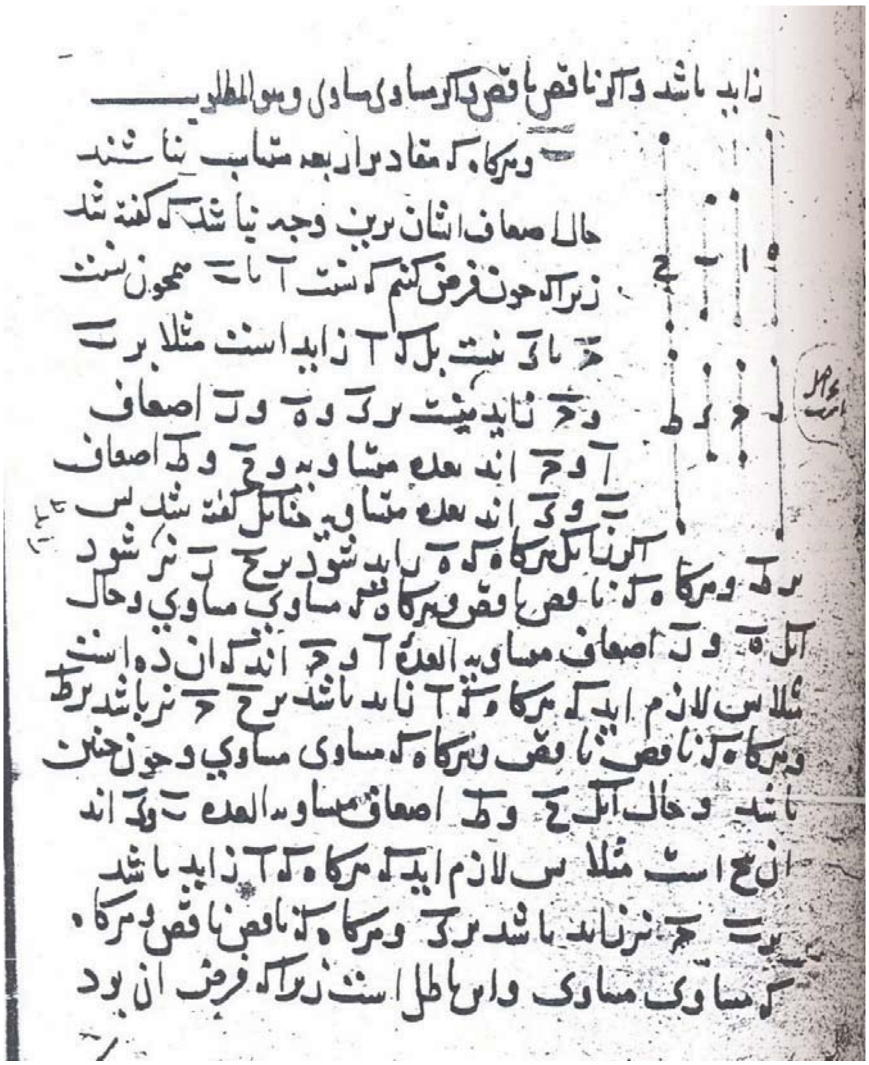

زاید باشد و اگر ناقص یا قص یا اگر ساوی ساوی وهو المطلوب
و هرگاه که مقادیر اربعه متناسب نباشند
حال اضعاف ایشان برین وجه نباشد که گفته شد
زیرا که چون فرض کنیم که نسبت آ با ب همچون نسبت
ج با د نیست بل که آ زاید است مثلا بر ب
و ج ناید نیست بر د و ه و ز اضعاف
آ و ج اند علی متساوی و ح و ط اضعاف
ب و د اند علی متساوی چنانک گفته شد پس
اگر نا آنک هرگاه که ه زاید شود بر ح ز نیز شود
بر ط و هرگاه که ه ناقص یا نقص و هرگاه که ه مساوی مساوی و حال
آنک ه و ز اضعاف متساوی العدد آ و ج اند که ان ده است
مثلا پس لازم اید که هرگاه که آ زاید باشد بر ب ج نیز باشد بر د
و هرگاه که آ ناقص یا نقص و هرگاه که مساوی مساوی و چون چنان
باشد و حال آنک ح و ط اضعاف مساوی العدد ب و د اند
از یک است مثلا پس لازم اید که هرگاه که آ زاید باشد
بر ب ج نیز زاید باشد بر د و هرگاه که ناقص یا قص و هرگاه
که مساوی مساوی و این باطل است زیرا که فرض ان بود

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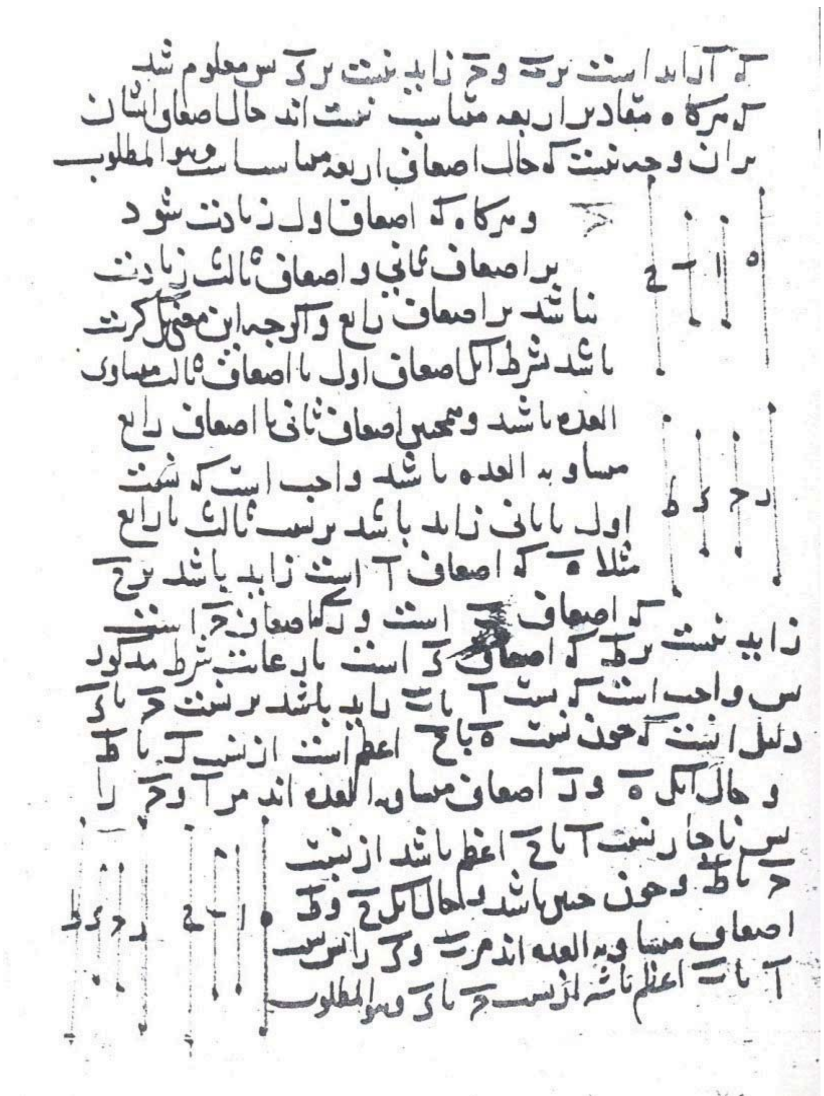
که آ با د است بر بج و حز اید نسبت بر کس معلوم شد
که هرگاه مقادیر اربعه متناسب نیست اند حال اضعاف ایشان
بر آن وجه نیست که حال اضعاف اربعه متناسبست وهو المطلوب
و هرگاه که اضعاف اول زیادت شود
بر اضعاف ثانی و اضعاف ثالث زیادت
نباشد بر اضعاف رابع و اگر چه این معنی کرت
باشد شرط آنکه اضعاف اول با اضعاف ثالث مساوی
العدد باشد و همچنین اضعاف ثانی با اضعاف رابع
مساوی العدد باشد واجب است که نسبت
اول با ثانی زاید باشد بر نسبت ثالث با رابع
مثلا ه که اضعاف آ است زاید باشد بر ح
که اضعاف ب است و ک که اضعاف ج است
زاید نیست بر ط که اضعاف د است با رعایت شرط مذکور
پس واجب است که نسبت آ با ب زاید باشد بر نسبت ج با د
دلیل آنست که چون نسبت ه با ح اعظم است از نسبت ک با ط
و حال آنکه ه و ک اضعاف مساوی العدد اند مر آ و ج را
پس ناچار نسبت آ با ح اعظم باشد از نسبت
ج با ط و چون چنین باشد و حال آنکه ح و ط
اضعاف متساوی العدد اند مر ب و د را پس
آ با ب اعظم باشد از نسبت ج با د وهو المطلوب

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