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Quṭb al-Dīn al-Shīrāzī's Appendix to Book I of his Persian Translation of Euclid: Text, Context, Influence

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Abstract
The first part of the paper describes the Persian translation of Naṣīr al-Dīn al-Ṭūsī's *Taḥrīr Kitāb Uqlīdis* by Quṭb al-Dīn al-Shīrāzī, with a primary focus on his appendix to book I. Part of a larger encyclopedic collection, al-Shīrāzī's translation continued to be read for centuries. As evidence of the work's influence, al-Shīrāzī's appendix to book I was included in a nineteenth century printed edition of Muḥammad Barakat's commentary on book I of al-Ṭūsī's treatise. I discuss this later use of al-Shīrāzī's appendix in the second part of the paper.

KeyWords: Quṭb al-Dīn al-Shīrāzī; Muḥammad Barakat; Naṣīr al-Dīn al-Ṭūsī; *Durrat al-Tāj*

Introduction to Part I: Quṭb al-Dīn al-Shīrāzī

Quṭb al-Dīn al-Shīrāzī (634/1236-710/1312), as so many scholars of his day, was a polymath.[1] Trained in medicine under his father, he initially practiced in Shīrāz, his native city. Seeking greater knowledge, he left his medical practice and traveled widely, searching out teachers and books. About 1262 he came to Marāgha where he was associated for several years with Naṣīr al-Dīn al-Ṭūsī, primarily devoting himself to mathematical sciences. He is known to contemporary historians of science for his contributions to non-Ptolemaic mathematical models describing planetary motions.

Leaving Marāgha, he traveled to Qazwīn to study philosophy. He felt deeply attracted to Aristotelian ideas as explicated by Ibn Sīnā and played a key role in reviving the peripatetic approach after the attacks of al-Ghazzālī. He sojourned in Baghdād for a time, then moved to Konya where he deepened the understanding of Ṣūfism he had acquired from his father. While in Anatolia, he began to participate in public life, serving as a judge in Sivas and Molatya. After several years, he returned to Tabriz and was commissioned by the Ilkhanid ruler as ambassador to the Mamlūk court in Egypt. During this period of public service he began to write extensively on philosophical topics.

After completing his diplomatic mission to Egypt, he returned to Tabriz and withdrew from public life, spending the last years of his life in seclusion, studying and writing mainly on philosophy and theology (although he also composed his famous commentary on Ibn Sīnā's *Qānūn* during this period, using books he had acquired in Cairo). He is reputed to have been a master chess player and skillful in playing the lute, occupations which he was able to indulge after he retired from public life.

Al-Shīrāzī's Translation: General Characteristics

I have, in another venue, discussed some of the more striking features of this translation (De Young, 2007). Because my earlier study has not circulated very widely internationally, I summarize its major findings in this section.

Al-Ṭūsī's *Taḥrīr Uṣūl Uqlīdis*, in addition to restating the basic Euclidean arguments of the *Elements* in a more streamlined Arabic form, had also included nearly two hundred notes inserted by al-Ṭūsī, most frequently introduced by the Arabic statement "I say" (*aqūlu*). On initial reading, this term seems to imply that these notes are the work of the author himself. Recent research reveals that this interpretation is

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1. These brief biographical notes are drawn from Nasr (1976, pp. 245-253).

incorrect.[1] These inserted notes include historical notices concerning differences between the two main Arabic translations of Euclid's treatise,[2] editorial notes reporting popular names for specific propositions, discussions of problematic mathematical points in Euclid's demonstrations, and alternative demonstrations for some propositions. Alternative demonstrations made up nearly half of the added material in al-Ṭūsī's treatise. One of the most striking features of al-Shīrāzī's translation is the removal of most of these alternative demonstrations from the text. Another victim of al-Shīrāzī's editing was the "demonstration'' of Euclid's parallel lines postulate (Daffaʾ, 1984, pp. 47-51; Jaouiche, 1986, pp. 201-226) which was replaced by a "shorter" demonstration (De Young. 2007, pp. 42-45). Al-Ṭūsī's extensive discussion of possible arrangement of triangles in proposition 47 has also been removed from the translation, as have most discussions of alternative positions for diagrams in book I.[3] Al-Shīrāzī does not discuss the rationale for this draconian editing of the treatise, although taken as a whole these modifications to the text seem consistent with a pedagogical concern.[4]

In other ways, too, the text has been edited in the process of translation. In general, the rendition of the geometrical arguments could be described as a close paraphrase, rather than a literal translation of the original Arabic. The technical vocabulary employed was the same Arabic terminology of al-Ṭūsī's text, while verbs, prepositions and adverbs were converted into Persian equivalents.[5] Several stereotypical

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1. Many of these mathematical comments were appropriated from earlier authors. Most of the alternative demonstrations, for example, were borrowed without ascription from the treatise *Fī Ḥall shukūk kitāb Uqlīdis fī'l-uṣūl* of Ibn al-Haytham (De Young, 2009b).
2. The traditional historiography of the Greek-Arabic transmission describes two translations, the first by al-Ḥajjāj ibn Yūsuf ibn Maṭar before the middle of the 2nd/8th century was later revised by the translator himself for caliph al-Ma'mūn. The second, by Isḥāq ibn Ḥunayn, was made near the end of the 2nd/8th century and revised by Thābit ibn Qurra at the beginning of the 3rd/9th century. It is from this final revision that all extant primary testimonia trace their origin. Close reading of the primary literature reveals that there have also been considerable crossovers from one version to another, resulting in several clearly distinguishable textual families. The situation is complex and the best summary of our current understanding has been given by Brentjes (2001, pp. 41-51).
3. These alternative positions are also ignored in the diagram that al-Shīrāzī added to summarize the content of book I. The diagram will be discussed in detail later in the paper.
4. It is also possible that al-Shīrāzī simply considered these notes unnecessary to the encyclopedic project within which this translation was made.
5. The retention of Arabic technical terminology is noteworthy in light of another example of translation of an Arabic mathematical text into Persian – the *Ketâb al-nejârat (Sur ce qui est indispensable aux artisans dans les constructions géometriques)* by Abu al-Wafâ al-Buzjâni. Aghayani-Chavoshi (2010) has recently edited the text of the earlier of two Persian translations and has compared it to the Arabic original. His linguistic analysis (pp. 39-75) indicates that the translator made a concerted effort to construct a distinctly Persian diction when translating, including use of archaic Persian terminology to create technical terms.

phrases used by al-Ṭūsī were rephrased in the Persian translation. None of these modifications are crucial to interpreting the text, but they suggest a freedom to manipulate the original source during the translation process. Translation is certainly not construed to be a mechanical substitution of a word in the original with a word in the target language. Other translations from Arabic into Persian –at least in the mathematical sciences– show a similar tendency to paraphrase rather than translate literally (De Young, 2009a).

Al-Shīrāzī has also introduced additions into al-Ṭūsī's treatise. For example, following the listing of the Euclidean axioms al-Shīrāzī introduces "demonstrations" of these axioms (De Young, 2007, pp. 31-47). While they fall short of full mathematical rigor, these added statements appear intended to convince readers that the postulates offer a believable description of mathematical reality. These "demonstrations" may have been borrowed from the *Iṣlāḥ* ("Correction") of the *Elements* composed by Athīr al-Dīn al-Abharī (d. 663/1265). Somewhat similar material also appears in the *Taḥrīr Uqlīdis* by the Pseudo-Ṭūsī (completed 698/1298), but it is formulated differently and presented in a different order (ibid, p. 31).[1] Al-Shīrāzī has also added his own mathematical notes in various places. Many of these are more extensive explanations of points made by al-Ṭūsī (ibid, pp. 54-75).

Among the additions to al-Shīrāzī's text, one of the more striking –at least visually– is the short appendix attached to book I. The remainder of my paper will focus on this addition, mentioned also by Brentjes (1998, p. 78), and the influence of this material in the Indian subcontinent. The appendix consists of a diagram (*shakl*) in which the translator has combined all the diagrams of book I into a single figure.[2] The diagram is accompanied by a textual description to explain which lines in the combined figure are needed to construct the diagram for each Euclidean proposition.[3] It seems highly probable that this appendix was the work of al-Shīrāzī himself because it appears in every

1. In the past, this redaction was sometimes considered to be an edited version of the genuine *Taḥrīr* of al-Ṭūsī, but the date of composition (698/1295) makes this impossible (Sabra, 1969, p. 18) and a close textual study of the two versions indicates that they differ in many important features (De Young, 2003; 2012a). These "demonstrations" are not found in the *Taḥrīr* of al-Ṭūsī.
2. Crozet (1999, pp. 132-140) discusses the important distinction between *shakl* (the geometric figure that exists in the world of mathematical entities) and *Ṣūrah* (the figure or picture constructed on a page). The geometer discusses specific mathematical aspects of the *shakl*, but uses the *Ṣūrah* constructed on the page to illuminate his thoughts on the geometric relations embodied in the *shakl*. The term *shakl* can also refer to the propositions to which the figures belong.
3. Persian text and English translation are given in the Appendix.

manuscript copy of the Persian translation that I have examined. It is also clear that the diagram and its accompanying explanatory text were not originally part of al-Ṭūsī's treatise since no manuscripts of the latter work include this appendix. The purpose of this combined diagram is not stated explicitly in the manuscripts of al-Shīrāzī's text, but the effort seems to be consistent with the hypothesis that al-Shīrāzī's intent was pedagogical –to aid students in reviewing or retaining the content of book I.[1]

Al-Shīrāzī's translation within *Durrat al-Tāj*

Al-Shīrāzī is credited with the earliest translation of Euclidean geometry into Persian. His translation was part of an encyclopedic project best known under the title *Durrat al-Tāj li-ghurrat al-Dubāj* (Pearl of the Crown for the outstanding Dubāj). The treatise, completed in 705/1305 near the end of his life, was dedicated to Dubāj ibn Ḥusām al-Dīn Fīl-Shāh ibn Sayf al-Dīn Rustam ibn Dubāj Isḥāqāwand, ruler of Bayah Pas in Gīlān province of Iran (Savage-Smith, 2005, p. 67).[2]

The complete treatise consists of an introductory essay (*fātiḥah*) discussing knowledge, in three sections (*faṣl*), followed by five major divisions (*jumlah*):

- Logic – divided into seven chapters (*maqālah*)
- Philosophy – divided into two parts (*fann*)
- Physics – divided into two parts (*fann*)
- Mathematical sciences – divided into four parts (*fann*): Euclidean geometry (an edited translation of al-Ṭūsī's Arabic redaction), Ptolemy's *Almagest* (a translation of the summary of the *Almagest* by ʿAbd al-Mālik ibn Muḥammad al-Shīrāzī), arithmetic, and music (mainly quotations from al-Fārābī, Ibn Sīnā, and ʿAbd-al-Muʾmin)
- Metaphysics – divided into two parts (*fann*)

It concludes with an appendix (*khātimah*) nearly as long as the work itself, divided into four sections (Quṭb) discussing dogmatic theology, religious law, practical philosophy, and mysticism.[3]

1. This composite diagram was explicitly connected to educational objectives when it was appended to the nineteenth century lithograph edition of the commentary on book I of al-Ṭūsī's treatise by Muḥamad Barakat. We shall consider this treatise and its pedagogical objectives below.
2. Brentjes (1998, p. 78) gives the date of composition as 1282 (or 680 AH). There is no justification presented, and manuscript evidence indicates a later date.
3. Parts of the treatise have been edited (Savage-Smith 2005, p. 68; Walbridge 1992, p. 358), the introduction and *jumlah* 1, 2, 3, 5 by Sayyid Muḥammad Mishkāt (Tehran, 1317-1320/1938-1941) and three *fann* of *jumlah* 4 by Sayyid Ḥasan Mishkān Ṭabasī (Tehran, 1317-1324/1938-1944). The *khātimah* was edited by Māhdukht Bānū Humāyī (Tehran, 1369/1991).

This philosophical encyclopedia has many parallels to the earlier scientific and philosophical encyclopedia of Ibn Sīnā (d. 428/1037), although the treatise is considerably less massive. The organization mirrors that used by Ibn Sīnā. The division of the subject material and the order of topics are the same. Al-Shīrāzī even employs the same terminology to name most of the divisions and subdivisions. The parallels are not surprising. Quṭb al-Dīn had studied the writings of Ibn Sīnā for many years and had been deeply influenced by the Aristotelian approach of Ibn Sīnā. Like the earlier encyclopedia, al-Shīrāzī's treatise is an exposition and interpretation of Aristotelian thought as seen through the lens of illuminationist philosophy.

In this paper, we are primarily concerned with the section on Euclidean geometry. Like Ibn Sīnā's *Kitāb al-Shifā'*, al-Shīrāzī's encyclopedia included a condensed version of Euclid's *Elements*. Al-Shīrāzī did not, however, follow the lead of Ibn Sīnā and construct his summary of Euclid directly on the Arabic translations from the Greek.[1] Rather, he translated (and edited) the Arabic classic, *Taḥrīr Kitāb Uqlīdis* of Naṣīr al-Dīn al-Ṭūsī (597/1201-672/1274), into Persian.[2] It is possible that the removal of al-Ṭūsī's mathematical notes (to be discussed in the next section) may have been influenced, at least in part, by a desire to condense the text for inclusion in the encyclopedia.

Because of its size, sections of the encyclopedia were sometimes copied out and circulated independently from the main work. Whether as part of the encyclopedia or as an independent treatise, al-Shīrāzī's Persian rendition of Euclid continued to play a role in the mathematical landscape of Persian-speaking areas for centuries, although its influence always remained subordinate to that of al-Ṭūsī's Arabic version.

Introduction to Part II: Muḥamad Barakat

Very little biographical information is available for Muḥamad Barakat. The title page of the printed edition of his commentary denotes him as "Allahabādī", indicating that he was from and/or worked in Allahabad,

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I have not seen these editions. I rely on manuscripts Tehran, Sanā 227; London, British Library, Add. 7695; Columbia University, Plimpton Or. 282.

1. The Arabic text of the geometrical section has been edited by Sabra (1976). Its relation to the Arabic primary transmission and geometrical content have been discussed by De Young (2002; 2012c).

2. Pourjavady and Schmidtke (2004, p. 313), suggest that al-Shīrāzī's translation is based on the *Taḥrīr Uqlīdis* by Muḥyi al-Dīn al-Maghribī (d. between 680/1281 and 690/1291). The rationale for this claim is not made explicit. They cite Mishkāt, the modern editors of portions of *Durrat al-Tāj* (see p. 6, note 2), but do not report the specifics of his argument. Recent study of the text (De Young, 2007) shows that it is a based on al-Ṭūsī's treatise.

a city some 50 km south of Lucknow, capital of the Indian principality of Oudh (or Awadh) and seat of the Farangi Mahall, an important center of Islamic learning. The Nawabs of Oudh were often generous patrons of scholars and attracted many important intellectuals to their court. The title of respect, "Mawlana", bestowed on him implies that he enjoyed some standing in society, but whether this prestige derived from anything more than intellectual ability is unclear.

Barakat is reported to have been active in the seventeenth century and to have enjoyed a reputation as a competent mathematician.[1] This reputation is apparently based almost completely on his commentary, since the modern bio-bibliographical literature does not credit him with any other mathematical or scientific writings (Rahman, 1982, p. 407). From the comments included in his treatise, we may deduce that he had interests in philosophy and logic as well as mathematics.

Muḥammad Barakat's commentary on al-Ṭūsī's *Taḥrīr*

Barakat's commentary is limited to book I of al-Ṭūsī's redaction. The reason for this limitation is not stated in the introduction to the commentary. But whatever Barakat's motives may have been, book I lays the foundation for much of the remainder of Euclid's treatise and so may be considered fundamental.[2] Moreover, once a student has learned how to read and understand the argumentation used in the demonstrations of book I, he should be able to peruse the remaining books by applying essentially the same analytical techniques. Perhaps it was just such a pedagogical outlook that motivated Barakat's commentary. In any case, it was the limited and well-defined focus of the commentary that made it attractive to the Farangi Mahall and its educational reformers of the eighteenth and nineteenth centuries.

The treatise which Barakat sought to explicate in his commentary was composed by Naṣīr al-Dīn al-Ṭūsī in 646. It included the entire thirteen books of the genuine *Elements* of Euclid, together with the apocryphal books XIV and XV, as had the original Arabic translations from the Greek.[3] After its composition, al-Ṭūsī's text rapidly took on a

1. He was familiar with mathematical scholarship, citing, among others, Shams al-Dīn al-Khafrī (d. 958/1551), an earlier commentator on al-Ṭūsī's *Taḥrīr* (Sezgin, 1974, p. 113; Saliba, 1994), and "commentators on *Ashkāl al-Taʾsīs*," an extract of 35 propositions drawn mainly from books I and II of the *Elements*. Presumably he is referring primarily to Qāḍīzāda al-Rūmī (d. about 840/1436) who penned a very popular commentary to the treatise (Sezgin, 1974, pp. 114-115; De Young, 2001).

2. Barakat is not alone in singling out book I. Ibn al-Haytham's *Kitāb fī Ḥall Shukūk Kitāb Uqlīdis*, for example, devoted approximately 40% of its bulk to book I. Many of the problematic points raised in Euclid's discussion appear first in book I.

3. Although the Arabs had consistently attributed both book XIV and XV to Hypsicles (about first century BC), modern scholarship usually credits him only with authorship of book XIV –

canonical status and was read and copied repeatedly until well into the nineteenth century. Al-Ṭūsī, who headed an institution devoted to research in mathematical cosmology and astronomy at Marāgha, apparently intended his treatise to form the foundation of mathematical studies that would culminate with the reading of Ptolemy's *Almagest.* It is probable that its role as foundational study for traditional mathematics education prompted Barakat to use this treatise as the foundation for his commentary.

Al-Ṭūsī appears to assume a readership that was intent on mastering the entire tradition of Greek mathematical thought with the ultimate goal of studying Ptolemy's mathematical cosmography in the *Almagest*. In order to read the *Almagest* intelligently, these students would need a thorough grounding in the Greek mathematical tradition, beginning with Euclid's *Elements*. In addition to his version of the *Elements*, al-Ṭūsī also prepared redactions of these so-called Intermediate Books (Steinschneider, 1865; al-Ṭūsī, 1939-1940; Aghayani-Chavoshi, 2005). The students studying these texts would not be intellectually immature or mere beginners in the field. They would already have mastered the basic disciplines, including philosophy.

Barakat seems to have envisioned a somewhat less sophisticated audience. Al-Ṭūsī spends little time on philosophical issues raised by the text. He assumes an audience concerned primarily with Euclidean mathematics as a foundation for higher mathematical studies, so his commentary focuses on mathematical issues. Barakat's commentary, on the other hand, devotes considerable attention to the logic of the argumentation. When a technical term is used, Barakat typically includes a note referring back to the definition of the term or, if it was not explicitly defined by Euclid, explaining its meaning. Many other comments refer students back to earlier propositions where specific results used by Euclid were originally demonstrated. These latter comments are reminiscent of the references to earlier propositions that occur so frequently in the Pseudo-Ṭūsī redaction published in Rome in 1594 (De Young, 2012a). From these characteristics we may deduce that Barakat's intended audience was less advanced mathematically and perhaps more intellectually immature, since he seems to assume that they need considerable help in navigating the arguments of Euclid.

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perhaps based on a now-lost work of Apollonius. Book XV is considered a compilation from several sources, one of which may be Isidorus of Miletus (6th century AD), who wrote a commentary that no longer survives (Vitrac and Djebbar, 2011, pp. 31-32).

Barakat's commentary in the Dars-i-Niẓāmī

Barakat's treatise appears to have been composed for students working in relative isolation and primarily interested in the deductive logical structure of the text. Yet a century later his work was adopted for use in the Islamic madrasas under a curricular reform usually known as the Dars-i-Niẓāmī. The curriculum is named for Niẓāmuddīn al-Sehalvi (d. 1748), who first proposed it in the middle of the eighteenth century. Niẓāmuddīn's father, Quṭb uddīn, a religious scholar of sufficient renown to have a personal acquaintance with Mughal Emperor Aurangzeb, had been killed in a property dispute. Aurangzeb took a personal interest in the case and deeded the abandoned Lucknow estate of a European trader (hence its name, Farangi Mahall) to the family in compensation. Niẓāmuddīn became a recognized religious scholar in his own right and Farangi Mahall developed into one of the most important centers of Islamic scholarship in India. It was noted especially for its liberal policies, accepting both Shīʿite and Sunnī students. Many of those who studied with its resident scholars carried the vision of the Farangi Mahall into the schools they established across the subcontinent (Robinson, 2001).

Niẓāmuddīn proposed his new curriculum as a response to an earlier proposal from Shah Walliullah of Delhi. A deeply devout and immensely learned religious scholar, Shah Walliullah (1703-1762) was distressed at the continued decline of the Islamic community in India. His response was to suggest that the training of the religious leadership needed to be buttressed with more study of religious sciences so that they could be better equipped to guide the religious life of the community. Niẓāmuddīn disagreed. His counter-proposal assumed that religious leaders already had a sufficient grounding in Islamic sciences. What was lacking was an understanding of science and philosophy so that religious leaders could appropriately apply Islamic learning to the practical problems of daily life in the modern world. Moreover, adding still more texts to an already lengthy curriculum meant that scholars were often too old by the time they completed their studies and no longer had the energy and drive necessary to provide effective leadership in the community. To that end, Niẓāmuddīn proposed to reduce the number of required religious texts, focusing only on what were most essential.[1] The remainder of the curriculum would concentrate on laying a firm foundation in science and philosophy, using short texts that taught the basic principles of the subjects, so that

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1. The basic texts included in Niẓāmuddīn's curriculum are outlined by Mujeeb (1985, pp. 407-408) and Desai (1978, pp. 14-15).

scholars would be able to develop deeper understanding as needed through independent reading and study.

In the mathematical sciences, Niẓāmuddīn's curriculum included five treatises:

- *Khulāṣat al-Ḥisāb*, an introduction to arithmetic by Bahāʾ al-Dīn al-ʿĀmilī
- *Sharḥ Taḥrīr Kitāb Uqlīdis*, the commentary on book I of al-Ṭūsī's treatise by Muḥammad Barakat
- *Tashrīḥ al-Aflāk*, by Bahāʾ al-Dīn al-ʿĀmilī
- *Risāla dar ʿIlm al-Hayʾa*, by ʿAlī Qushjī
- *Sharḥ al-Mulakhkhaṣ fī al-Hayʾa al-Basīṭa*, by ʿAlī al-Jurjānī

The first two treatises introduced the principles of arithmetic and geometry, while the last three focused on elementary astronomy and cosmography. The basic features of this mathematical curriculum mirrored the stages of traditional education in mathematical sciences, but replaced the ponderous Greek classics with shorter, easier introductions. The aim was clearly not to produce mathematicians so much as to provide a basic grounding in the mathematical sciences sufficient for scholars to read and understand religious texts and to be able to solve basic everyday problems in their communities.

The specific content of the Dars-i-Niẓāmī continued to evolve after the death of Niẓāmuddīn. The emphasis on rational sciences at the expense of religious sciences initially scandalized the religious community and unleashed a storm of criticism. In the succeeding decades, the scientific and philosophical content was steadily eroded, to be replaced by religious treatises. By the beginning of the nineteenth century, only the first two treatises seem to be regularly included in madrasa study. The *Khulāṣat al-Ḥisāb* appears more popular than Barakat's commentary on Euclid since there are many more surviving manuscript copies of the arithmetic text. Many of these manuscripts are produced with exceptionally widely spaced lines, suggesting that they might have been intended to allow students to insert interlinear notes (De Young, 1986). Moreover, the text was the subject of numerous commentaries and super-commentaries, both in Arabic and in Persian, which are still extant in Indian manuscript collections (De Young, 1995, p. 146). Many of these manuscripts have been heavily annotated by readers.

There are far fewer manuscripts of Barakat's commentary extant, and most copies that I have seen contain few marginalia or interlinear notes other than corrections inserted by the copyist. Thus, even though the treatise remained a recognized element in the curriculum, it appears that it was often quietly ignored in practice. We may therefore find it somewhat surprising that it was Barakat's commentary that was printed in the nineteenth century, while the *Khulāṣat al-Ḥisāb* apparently was never printed in India.[1]

Uniting Barakat's commentary and al-Shīrāzī's appendix

To Barakat's commentary, the editors of the lithograph edition joined a short Persian extract from al-Shīrāzī's version of al-Ṭūsī's *Taḥrīr* –the appendix to book I. This short segment (pages 95-96 in the lithograph edition) is the only Persian text in the printed treatise. The section begins with a half-page introduction entitled "Khātima al-Ṭabʿa" (epilogue of the printer).

The addition is introduced by an Arabic title delimited above and below by a single line and written in larger script: "This is what the author (ṣāḥib) of *Durrat al-Tāj Ġurrat al-Dubāj* said."[2] It is followed by the introductory statement of al-Shīrāzī's appendix, also delimited by horizontal lines. The remainder of the page is taken up with the diagram. The accompanying explanatory text occupies the second page.

This appendix was not included in the original treatise of Barakat.[3] Its inclusion in the printed version indicates that the editors must have considered it to be accessible linguistically and useful mathematically to the intended audience. And, as we have already stated, that intended audience seems to have been novices in the study of Euclidean geometry or at least students whose lack of preparation required that they be constantly reminded of the logic of the demonstrations through insertion of references to earlier propositions or definitions.

Concluding thoughts

Although Persian was the language of political administration and literature in the region of Oudh and Islamic India, Barakat composed his commentary in Arabic. This can only mean that Arabic remained the primary language for learning and discussing mathematics during this period. And although several Persian translations of classical Arabic mathematical works are known, they often seem to be subordinate to the Arabic originals. Barakat was not alone in using Arabic to write mathematics during this period. More than a century

1. A search of the WorldCat website found no record of any book bearing the title *Khulāṣat al-Ḥisāb*.

2. The fact that al-Shīrāzī is not identified by name is not significant. The editors apparently assume that his encyclopedia will be so readily familiar to readers that the author's name is superfluous.

3. At least it is not found in the manuscript copies I have seen: Aligarh, Mawlana Azad University Library, Jawahir 295 and Abdul Hai 680/57. The title of the section in the printed edition also implies that it has been introduced by the editor/printer.

later, Nawab Tafazzul Hussein Khan (1727-1800) translated Newton's *Principia* (1789), as well as other classic European mathematical works, from Latin into Arabic (Schaffer, 2009). And when al-Ṭūsī's *Taḥrīr* was printed in Tehran in 1298/1880, it was the Arabic original, not one of the Persian translations, that was produced.

Inclusion of al-Shīrāzī's figure and its supporting text in this printed edition carries implications for how the editors looked at the relation between visual and textual elements in the study of geometry. Although the text is clearly necessary to understand the intent of the figure, the diagram itself is apparently seen as important to the review or recapitulation of book I. It is placed at the end of the commentary text and refers back to or recapitulates the diagram of each of the textual propositions.[1] Netz (1998, pp. 37-38) has suggested that in classical Greek mathematics, the diagram stands as a metonym or substitute for the proposition. (Today many would probably regard the text as the primary form of the proposition.) From this point of view, the printers seem to regard al-Shīrāzī's figure as a metonym for all the propositions of book I.

Al-Shīrāzī's appendix to book I was in one sense reborn or took on a new life when it was added to the printed edition of this textbook. Both Muḥamad Barakat's commentary and al-Shīrāzī's diagram were based on the same treatise –the *Taḥrīr Kitāb Uqlīdis.* Both had features that made them attractive in an educational enterprise. Both appear to have been intended for beginning or relatively unsophisticated students.

The publication of Barakat's textbook, with its appendix from al-Shīrāzi, was apparently intended to strengthen the Indian Islamic community to enable it to resist pressures from the British colonial agenda, which often included teaching new European approaches to geometry and mathematics. But by looking backward instead of forward, by turning to a text that had been the pinnacle of mathematics education centuries earlier, the Islamic community lost its chance (if it really had a chance) to propose an offer as alternative to modern mathematics taught in British colonial schools. Nevertheless, this effort to create a geometry textbook for use in the Islamic madrasas shows how long-lived was the tradition of Euclidean mathematics deriving from the canonical work of al-Ṭūsī.

1. In Arabic manuscripts, geometrical diagrams are often placed at the end of the proposition, following the demonstration (De Young, 2012b). Al-Shīrāzī's placement of this composite diagram is consistent with that tradition.

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APPENDIX

In this appendix, I translate the Persian text of al-Shīrāzī’s addition to book I into English. This translation is followed by the Persian text. The text is transcribed from the printed edition, but has been compared to the available manuscript sources.

When editing the diagram, I have retained the Arabic/Persian labeling. In general, I follow the Library of Congress conventions for transliterating Persian labels into English although I have introduced a few modifications as indicated in the table.

ع	س	ن	م	ل	ك	ي	ط	ح	ز	و	هـ	د	ج	ب	ا
O	S	N	M	L	K	Y	Ṭ	Ḥ	Z	W	E	D	G	B	A
كج	كب	كا		غ	ظ	ض	ذ	خ	ث	ت	ش	ر	ق	ص	ف
X	P	J		Ġ	Ý	Ḍ	Ḏ	Ḫ	Ŧ	T	Ŝ	R	Q	Ṣ	F

Table 1: Conversion of Persian labels to Roman script. The diagram is complex and requires more letters than exist in the Arabic alphabet. It is interesting that al-Shīrāzī did not use letters from the Persian alphabet, but instead opted to use double letters from the Arabic alphabet to label the points. The letters are presented here in their typical *abjad* or alphanumeric sequence rather than the lexical order found in a modern Arabic dictionary.

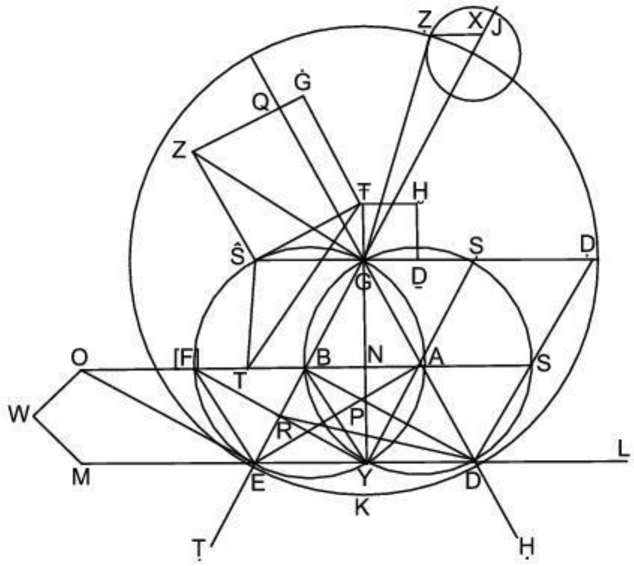

Figure 1: Al-Shīrāzī’s figure, edited from British Library, add. 7695, folio 18a. The copyist has inadvertently omitted letter F from the diagram. The diagram itself has been carefully constructed.

English Translation

I say: It is possible to combine the totality of the diagrams of this book into one diagram, picturing them in this way. And therefore, since it is for beginners, I give an indication for each diagram from which lines they arise.

(1) From circle GBY and [circle] GAY and triangle AGB.

(2) From circle DKE and [from] one of the two [previously] mentioned circles and triangle GAB and the sought line should be either AD or BE.

(3) From one of the two [previously] mentioned circles and either AD or BE should be cut off from AḤ or BṬ.

(4) From [triangles] GDY [and] GEY.

(5) From lines ḤG, GṬ, AB, DB, EA.

(6) From triangle DGE with [either] one of [lines] DB, AE.

(7) From the quadrilateral ABED and [lines] DB, EA.

(8) From triangle GAB with [triangles] ADE, BED.

(9) From [lines] ḤG, GṬ and triangle AYB and line GY.

(10) From triangle AGB and [line] GN.

(11) From [line] SO and [triangle] AGB.[1]

(12) From circle DKE and [line] LM and [triangle] GDE and [line] GY.

(13) From [line] DE and [lines] YA, YN.

(14) From [line] ḤA and [lines] DY, DB.[2]

(15) It can arise from many [lines].

(16) From triangles BYE, FYE and lines EṬ, EM.

(17-20) It can arise from many [triangles and lines].

(21) From triangle GDB and lines GN and BN.

(22) From the larger circle and circle XÝ and lines XÝ, ÝG.

(23) It can arise from many [lines].[3]

(24) From quadrilateral ABDE with triangle ABG.

1. This is the formulation in the text. I believe the diagram would be clearer if it included also line GN as in the previous proposition.

2. Neither DY nor DB is perpendicular to line ḤA. The typical Euclidean manuscript would have shown one of the lines as perpendicular. Al-Shīrāzī could have used the same lines as in proposition 13. It is unclear why he chose different lines in this case.

3. This statement is omitted from British Library, add. 7695. If al-Shīrāzī is thinking only of lines and angles mentioned in the enunciation, his statement is correct. But to construct an equivalent to the given angle, we must use two intersecting circles. There are only two pairs of intersecting circles and only circles GAY, GBY are appropriate to the diagram requirements, sothere are really only two triangles possible –triangle GAB or triangle YAB although several angles could be used for the given angle. Because these two circles are constructed to have equal radii, the diagram they produce will be “overspecified”. (The term was introduced by Saito (2006, p. 82) to describe diagrams which portray a more limited case than is required in the statement of the proposition. Overspecification is common in manuscripts.)

(25) It can arise from many [triangles / lines].
(26) From triangles BFE, GDE and lines GY, DB.
(27, 28, 29) From [lines] SO, LM with [line] GBEṬ, for example.
(30) From [lines] ḌD, ṢY, GE [and] SAB.
(31, 32, 33, 34) It can arise from many [lines].[1]
(35) From areas ADYB and BDYF.
(36) From areas ADYB, BYEF, BDYF.
(37) From area BYEF with [lines] BE, FY and triangle BAY and [line] OE.[2]
(38) From area ASDY and diameter AD and area BFEY and diameter EB.[3]
(39) From quadrilateral ABYD and diameters AY, DB and lines AP, PE.[4]
(40) From triangles SDA, FEB and lines SF, DE and lines DR, RF.
(41) From area AYEB and triangle FYE and lines AF, YE.
(42) From triangle GDE [and area] ṢḌDY.[5]
(43) From area ḌGDE and diameter DG[6] and lines SAB, ṢAY.[7]
(44) From the described figure (43) with triangle BEF and angle FEO, for example.
(45, 46) It can arise from many lines.
(47) From triangle ŦGŜ[8] and squares ŦD̲, ŜN, ŜĠ and lines GQ, GZ, ŜT.
(48) From triangle AGB and perpendicular GN.[9]

And likewise it is possible that the diagrams of the other books may be combined into one figure following an analogous method.[10]

1. The number 32 is accidentally omitted from this statement in British Library, Add. 7695.
2. The copyists of British Library, Add. 7695 and Sana 227 have written "FOE" instead of "and [line] OE".
3. The copyist of Sana 227 has written "EY" instead of "EB."
4. The copyist of Sana 227 has written "ADY, DBY" instead of "AP, PE".
5. The copyist of British Library, Add. 7695 has written "ṢDY" instead of "[and area] ṢḌDY".
6. The copyist of British Library, Add. 7695 inserts here "and line DG" which is not found in the other manuscripts consulted or in the printed edition.
7. The copyist of British Library, Add. 7695 has "SAY" instead of "ṢAY".
8. The copyist of British Library, Add. 7695 has written "ŦŜ".
9. Al-Shīrāzī could also have used triangle GDE and perpendicular GY.
10. The copyists of Sana 227 and Plimpton or. 282 omitted this summary statement.

Persian Text

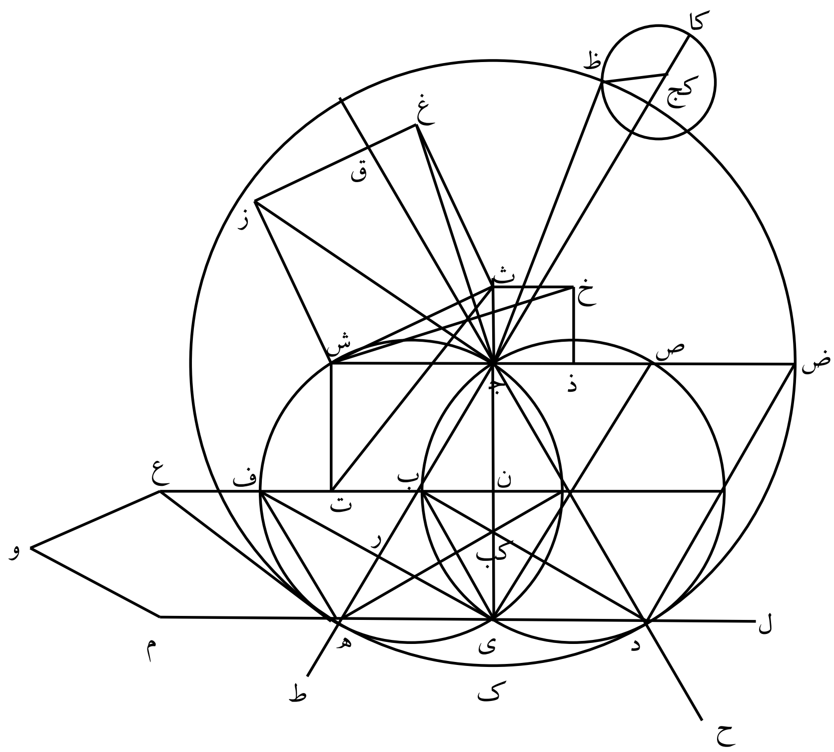

Figure 1: Al-Shīrāzī’s figure as edited from the printed commentary of Muḥammad Barakat, p. 94. The main part of the diagram has been considerably compressed, making the figure more difficult to read. The letters K and S have been inadvertently omitted by the copyist. Placement of the labels is generally consistent with placement in the print edition except that label Y has been moved outside the circle for clarity (the location would normally be occupied by label K). Some of the labels are relatively far from the points to which they belong (labels A and Y, for example). The circles have become decidedly elliptical and some lines that should be parallel are obviously not so in the diagram. The reason for these lapses is unclear. Perhaps they reflect carelessness by the copyist or perhaps they are an artifact of the lithograph process itself.

The Persian text has been transcribed from the printed version of Muḥammad Barakat’s commentary (pp. 94-95), although the text has been compared to the manuscripts available to me. Variants have been noted in the English translation. In Arabic and Persian manuscripts, letters designating geometrical entities are usually indicated by a line over the letter. In this transcription, these letters are enclosed in parentheses.

و من می‌گویم ممکن است که جملهٔ اشکال این مقالت را در یک شکل تصویر کنند برین وجه و به‌جهت آنکه تا بر مبتدیان آسان باشد اشارت کنیم که هر شکلی از کدام خطوط برخیزد.

و اما ا از دائرهٔ (ج ب ی) و (ج ا ی) و مثلث (ا ج ب).

و اما ب از دائرهٔ (د ک هـ) با یکی از دو دائرهٔ مذکور و مثلث (ج ا ب) و خط مطلوب یا (ا د) باشد یا (ب هـ).

و اما ج از یکی از دو دائرهٔ مذکور و (ا د) یا (ب ه) مفصول باشد از (ا ح) یا (ب ط).

و اما د از (ج د ی) و (ج هـ ی).

و اما هـ از خطوط (ج ح) و (ج ط) و (ا ب) و (د ب) و (هـ ا).

و اما و از مثلث (د ج هـ) با یکی از (د ب) و (هـ ا).

و اما ز از ذو اربعة اضلاع (ا ب هـ د) و (د ب) و (هـ ا).

و اما ح از مثلث (ج ا ب) با (ا د هـ) و (ب هـ د).

و اما ط از (ح ج) و (ج ط) و مثلث (ا ی ب) و خط (ج ی).

و اما ی از مثلث (ا ج ب) و (ج ن).

و اما یا از (س ع) و (ا ج ب).

و اما یب از دائرهٔ (د ک هـ) و (ل م) و (ج د هـ) و (ج ی).

و اما یج از (د هـ) و (ی ا) و (ی ن).

و اما ید از (ح ا) و (د ی) و (د ب).

و اما یه از بسیاری برمی‌خیزد.

و اما یو از مثلث (ب ی هـ) و (ف ی هـ) و خط (هـ ط) و (هـ م).

و اما یز و یح و یط و ک از بسیاری خطوط برمی‌خیزد.

و اما کا از مثلث (ج د ب) و خط (ج ن) و (ب ن).

و اما کب از دائرهٔ بزرگ و دائرهٔ (کج ظ) و خط (کج ظ) و (ظ ج).

و اما کج از بسیاری برمی‌خیزد.

و اما کد از ذو اربعة اضلاع (ا ب ه د) با مثلث (ا ب ج).

و اما که از بسیاری برمی‌خیزد.

و اما کو از مثلث (ب ف هـ) و (ج د هـ) و خط (ج ی) و (د ب).

و اما کز و کح و کط از (س ع) و (ل م) با (ج ب هـ ط) مثلاً.

و اما ل از (ض د) و (ص ی) و (ج هـ) و (س ا ب).

و اما لا و لب و لج و لد از بسیاری برمی‌خیزد.

و اما له از سطح (ا د ی ب) و (ب د ی ف).

و اما لو از سطوح (ا د ی ب) و (ب ی هـ ف) و (ب د ی ف).

و اما لز از سطح (ب ی هـ ف) با (ب هـ) و (ف ی) و مثلث (ب ا ی) و (ع هـ).

و اما لح از سطح (ا س د ی) و قطر (د ا) و سطح (ب ف هـ ی) و قطر (هـ ب).

و اما لط از ذو اربعة اضلاع (ا ب ی د) و قطر (ا ی) و (د ب) و خط (ا کب) و (کب ی).

و اما م از مثلث (س د ا) و (ف هـ ب) و خط (س ف) و (د هـ) و خط (د ر) و (ر ف).

و اما ما از سطح (ا ی هـ ب) ومثلث (ف ی هـ) و خط (ا ف) و (ی هـ).

و اما مب از مثلث (ج د هـ) و (ص ض د ی).

و اما مج از سطح (ض ج هـ د) و قطر (د ج) وخط (س ا ب) و (ص ا ی).

و اما مد از همین سطح با مثلث (ب هـ ف) وزاویهٔ (ف هـ ع) مثلاً.

و اما مه و مو از بسیاری برمی‌خیزند.

و اما مز از مثلث (ث ج ش) ومربعات (ث ذ) و (ش ن) و (ش غ) و خطوط (ج ق) و (ج ز) و (ش ت).

و اما مح از مثلث (ا ج ب) و عمود (ج ن).

و همچنین ممکن است که دیگر اشکال مقالات را در یک شکل جمع برین قیاس اگر کسی خواهد کند